

## MATH 240: Rosen §1.5 #14b sample solution

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14b. Every student in this class plays some sport.

- Let  $\text{Plays}(x, y)$  represent the predicate “student  $x$  plays sport  $y$ ”.
- Let  $C$  represent students in this class, and let  $S$  represent all sports.

Then we can formally encode the given statement as

$$\forall s : C. \exists t : S. \text{Plays}(s, t).$$

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(End of sample solution, beginning of general commentary.) Notice how the word “every” turns into a  $\forall$ , and “some” turns into an  $\exists$ .

By way of contrast, here is a solution which is logically correct, but unsatisfactory: let  $PS(x)$  represent the predicate “student  $x$  plays some sport”, and  $C$  represent students in this class. Then we can encode the statement as

$$\forall s : C. PS(s).$$

The reason this is unsatisfactory is that the predicate  $PS(x)$  represents a concept which is too complicated: it is hiding an existential quantifier. However, this could be a great *start* to a solution: once we have written this down we can then think about how to define the predicate  $PS(x)$  in terms of simpler predicates. We might then ultimately end up with something like the first solution above.