Discrete Math Exam 1

Friday, March 3, 2023

The exam will take place in class on Friday, March 3. You are **not** allowed to bring any notes, textbooks, or any other resources with you to the exam, and you may not use a phone (not even as a calculator). Bring only something to write with. I will provide a fresh copy of the exam with space for writing your solutions. I will also make scratch paper available if you need it. You may use a calculator, but it probably won't help.

This exam has two parts: a "take-home" part and an "on sight" part.

- For the "take-home" part, the questions shown here are the exact same questions you will see on the exam. You may prepare your solutions ahead of time using **any resources** including textbooks, other students and professors, previous quizzes and homeworks, or any sources on the Internet. You may also ask me for feedback on potential solutions, though I will not give hints for exam questions. Of course, I am happy to answer general questions, go over homework problems, or answer clarifying questions about exam problems.
- For the "on sight" part, you will not have access to the problems in advance. However:
  - The problems will be similar to problems you have done on homework assignments. The only difference is that some problems may involve synthesizing multiple concepts or skills from the course so far instead of only focusing on one concept or skill.
  - You should create practice problems and submit them, so I can share them with the class!
  - I may provide some additional practice problems as well.

Topics covered by the exam:

- Propositional logic
  - Boolean connectives $\wedge$, $\vee$, $\neg$, $\rightarrow$
  - Truth tables
  - Quantifiers $\forall$, $\exists$
  - Logic equivalence laws; proofs via sequences of equivalence
  - Translating English statements into formal logic

- Proof writing: proof techniques corresponding to propositional logic
- Set theory
  - Special sets $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$
  - Set comprehension notation, empty set, subsets
  - Set operations $\cap$, $\cup$, $-$

Take home problems

1. Prove that for all integers n, if n^2 is even then n is also even.

   Note: you may take as given the definitions of even and odd that we discussed in class, and you may assume that $\neg\text{Even}(n) \equiv \text{Odd}(n)$. *However, for the purposes of this problem, you may* not *cite any theorems we proved as examples in class (though you may certainly study them). You must give a complete proof from first principles, based on the definitions of even and odd.*

2. For each of the following two statements, you should either give a proof if it is true, or give a counterexample if it is false.

   (a) For all sets S, T, and A, if $S \subseteq T$ then $(A \cap S) \subseteq (A \cap T)$.

   (b) For all sets S, T, and A, if $(A \cap S) \subseteq (A \cap T)$ then $S \subseteq T$.