MATH 240 Module 6: Divisibility

due Friday, 17 March 2023

Learning Goals

- Understand and apply the divisibility relation
- Use the division algorithm to calculate quotients and remainders, and use quotient and mod operations appropriately to solve problems
- Prove properties of modular equivalence, and solve modular equivalences

Submission

You should submit:

- A PDF with your answers to the exercises (you may either type your answers and export as a PDF, or write your answers by hand and scan them using an app such as GeniusScan or CamScanner).
- You should also complete the mid-semester survey (`https://forms.gle/X5J5aayiMvm28pCM7`) and **include a statement in your PDF** that you have completed the survey.

Exercises

Exercise 1 Complete two more parts of the theorem from class about properties of the divisibility relation. Let a, b, and c be arbitrary integers.

(a) Prove that if $a \mid b$, then $a \mid bc$.

(b) Prove that if $a \mid b$ and $b \mid c$, then $a \mid c$.

Exercise 2 For each part, compute the quotient q and remainder r (according to the *division algorithm* from class) when a is divided by b.

(a) $a = 44$, $b = 8$

(b) $a = 777$, $b = 21$

(c) $a = -123$, $b = 19$

(d) $a = -1$, $b = 23$

Exercise 3 Consider taking a $r \times c$ square grid and numbering the cells consecutively row by row, starting with cell 0. For example, if we have a 3×8 grid, we would number the cells like this:

0	1	2	3	4	5	6	7
8	9	10	11	12	13	14	15
16	17	18	19	20	21	22	23

There are now two different ways we could identify a particular cell: by the number it contains, or by its (row, column) coordinates. For example, the cell labelled 19 in the grid above is at row 2, column 3 (note that we start numbering rows and columns at 0, so the above example has rows 0–2 and columns 0–7).

(a) Explain how we can convert from (row, column) coordinates to cell number. That is, given an $r \times c$ grid numbered according to the above scheme, how can we compute which number will be in cell (i, j)?

(b) Now explain how to convert in the other direction. That is, given a particular cell number n, how can we compute the row and column coordinates (i, j) at which we will find n?

Exercise 4 Prove that equivalence modulo m is a *congruence with respect to multiplication*. That is, prove that for all integers a, b, c, d and positive integers m, if $a \equiv_m b$ and $c \equiv_m d$, then $ac \equiv_m bd$.

Exercise 5 Solve each of the following modular equivalences for x. Your solution should be of the form $x \equiv_m k$ where $0 \le k < m$.

(a) $2x + 5 \equiv_7 3x - 12$

(b) $172x + 99 \equiv_{19} 359$