MATH 240 Module 3: Proofs

due Friday, 17 Feb 2023

Learning Goals

- Translate English statements to formal propositional logic, and use their structure to write informal proofs.
- Correctly reason using proof techniques appropriate to each logical connective.

Submission

You should submit:

- a PDF with your answers to the exercises (you may either type your answers and export as a PDF, or write your answers by hand and scan them using an app such as GeniusScan or CamScanner).

Exercises

When asked to prove a proposition:

- Translate the given proposition into formal propositional logic, using quantifiers as appropriate.
- Write a proof of the proposition.

Exercise 1 Prove: for all integers m and n, if mn is even, then either m is even or n is even (or both).

Exercise 2 Prove: for any positive integer n, n is even if and only if $7n + 4$ is even.

Exercise 3

(a) Prove that the sum of a rational number and an irrational number must be irrational.

(b) Prove that the product of a nonzero rational number and an irrational number must be irrational.

Exercise 4

(a) Give an example showing that it is possible for the sum of two irrational numbers to be rational.

(b) Prove that $\sqrt{2} + \sqrt{3}$ is in fact irrational.

Hint: simplify $(\sqrt{2} + \sqrt{3})^2$ and use the results from the previous exercise to show that $(\sqrt{2} + \sqrt{3})^2$ must be irrational; then explain why this shows $\sqrt{2} + \sqrt{3}$ must be irrational as well. In class, we proved that $\sqrt{2}$ is irrational. You may assume that $\sqrt{3}$ is also irrational. In fact, $\sqrt{n}$ is always irrational whenever n is not a perfect square, though we won't be able to prove this until later in the course.