MATH 240 Module 1: Propositional logic

due Friday, 3 Feb 2023

Learning Goals

- Evaluate propositional logic expressions.
- Translate English sentences into formal propositional logic notation.
- Construct truth tables for complex propositions.
- Use logical equivalences to manipulate propositions.

Submission

You should submit two files:

- A PDF with your answers to the exercises (you may either type your answers and export as a PDF, or write your answers by hand and scan them using an app such as GeniusScan or CamScanner). Note that you **must** submit a PDF! Submissions in any other format (`.docx`, `.pages`, ...) will need to be resubmitted. If you are not sure how to create a PDF document please ask for help!
- A completed version of `module1.disco`; see `https://replit.com/@BrentYorgey/Discrete-Math-Module-1`.

> **Feel free to use Disco** to help you complete the exercises!

Exercises

Exercise 1

Let the propositional variables p, q, and r be defined as follows:

$$p = \text{Unicorns are real.}$$
$$q = \text{Dragons are real.}$$
$$r = \text{Dr. Yorgey likes math.}$$

Using these variables, translate each of the following English sentences into formal propositional logic notation. For example, the sentence "Unicorns and dragons are real" could be translated as $p \wedge q$.

(a) Dr. Yorgey likes math, but unicorns are not real.

(b) Unicorns and dragons are real, and Dr. Yorgey likes math.

(c) Unicorns are real if Dr. Yorgey doesn't like math.

(d) Either dragons are real and Dr. Yorgey likes math, or unicorns are real and Dr. Yorgey doesn't like math.

(e) If either unicorns or dragons are real, then Dr. Yorgey likes math.

(f) Dragons either are or aren't real.

Exercise 2 Suppose that p and q are true propositions, and r is false. Evaluate whether each of the following propositions is true or false, and check your answers using Disco. For example, to evaluate $p \wedge r$ you could write

```
let p = true, r = false in p /\ r
```

at the Disco prompt.

Note: of course you could just evaluate each proposition using Disco without trying to evaluate it by hand first; but you can bet that you will have to do this by hand on an exam!

(a) $p \wedge (q \vee r) \wedge \neg r$

(b) $(r \to q) \vee (q \to r)$

(c) $((r \to r) \to r) \to r$

(d) $(p \vee \neg p) \wedge (q \vee \neg q) \wedge (r \vee \neg r)$

Exercise 3 For each of the following, *either*:

- use a truth table to show that the two expressions are logically equivalent for all possible truth values of the propositional variables, *or*
- give an example of specific truth values for which the two expressions are different.

1. $(Q \to \neg P) \stackrel{?}{\equiv} (P \leftrightarrow Q)$
2. $(P \vee Q) \to R \stackrel{?}{\equiv} (P \to R) \wedge (Q \to R)$
3. $(P \to Q) \to R \stackrel{?}{\equiv} P \to (Q \to R)$

Exercise 4 Each of the following is an **incorrect** attempt to establish a logical equivalence via a series of transformations. For each one, explain the mistake, and give specific values of the propositional variables for which the starting proposition does not have the same truth value as the ending proposition.

(a)
$$\begin{aligned} & p \vee (p \wedge q) \\ \equiv \quad & \{ \text{ Associativity } \} \\ & (p \vee p) \wedge q \\ \equiv \quad & \{ \text{ Idempotence } \} \\ & p \wedge q \end{aligned}$$

(b)
$$\begin{aligned} & p \wedge (\neg p \vee q) \\ \equiv \quad & \{ \text{ Distributivity } \} \\ & (p \vee \neg p) \wedge (p \vee q) \\ \equiv \quad & \{ \text{ Excluded Middle } \} \\ & T \wedge (p \vee q) \\ \equiv \quad & \{ \text{ Identity } \} \\ & p \vee q \end{aligned}$$

(c)
$$\begin{aligned} & \neg(q \vee (\neg p \wedge r)) \\ \equiv \quad & \{ \text{ De Morgan } \} \\ & \neg q \vee \neg(\neg p \wedge r) \\ \equiv \quad & \{ \text{ Double negation } \} \\ & \neg q \vee (p \wedge r) \end{aligned}$$

Exercise 5 Use a sequence of logical equivalences to prove each equivalence.

(a) $\neg(\neg(p \wedge p) \wedge \neg(q \wedge q)) \equiv p \vee q$

(b) $\neg p \to (q \to r) \equiv q \to (p \vee r)$

(c) $(p \vee q) \to r \equiv (p \to r) \wedge (q \to r)$

(d) $p \wedge (p \vee q) \equiv p$

Disco

You must also complete Disco programming **Exercises D1 and D2**: see `https://replit.com/@BrentYorgey/Discrete-Math-Module-1`.