Example: soup schedules.

- 10 soups
- want to pick 4 to eat

nPr

\# of schedules : $10 \cdot 9 \cdot 8 \cdot 7$ $\left(= \frac{10!}{6!} \right)$

- But what if we don't care about the order?

How many different <u>subsets</u> of 4 soups are there?

$10 \cdot 9 \cdot 8 \cdot 7$ is too big — counts different schedules that we want to consider the same.

2 sequences of 4 soups are "the same" if they are just permutations / rearrangements of each other. We want <u># of equivalence classes.</u>

A given set of 4 soups can be put in 24 orders — so gets counted exactly 24 times.

Hence, the number of different subsets is $\frac{10 \cdot 9 \cdot 8 \cdot 7}{24}$.

<u>Division rule</u>

Suppose we have n total choices, but we want to think of some of them as equivalent. If the choices always come in groups of exactly k that are equivalent (ie. each equivalence class has cardinality k), then the number of "actually different" choices (ie. # of equivalence classes) is n/k.

Ex. How many ways are there to seat 5 people around a circular table, if we only care who is sitting to the L/R of who?

5! ways to seat people if we do care who is in what seat.

But any given arrangement has a total of 5 equivalent arrangements which are all possible rotations.

Hence, by division rule, # of arrangements we consider different is

$$\frac{5!}{5} = \underline{4!} = 24.$$

Ex. Same situation but now we only care who is next to who.

Now we can mirror the arrangement — so we divide by 2

$24/2 = 12$ arrangements.

Ex. We have a set of size n, want to pick a subset of size k. How many ways can we do this?

$$n \cdot (n-1) \cdot (n-2) \cdots (n-k+1) = \frac{n!}{(n-k)!}$$

ways to pick k things in a specific order. But there are $k!$ ways to rearrange any set of k things.

Hence, the number of <u>subsets</u> of k things, ie not caring about order, is

$$\frac{n!}{(n-k)!} \Big/ k! = \frac{n!}{k!(n-k)!} = \binom{n}{k}$$

This is a "binomial coefficient" — "n choose k"

Ex. How many 8-bit binary strings are there with exactly 3 ones (and 5 zeros)?

This is counting how many ways we can pick a subset of 3 out of the 8 bit positions to put a 1.

$$\binom{8}{3} = \frac{8!}{3!(8-3)!} = \frac{8\cdot7\cdot6\cdot\cancel{5!}}{3!\cdot\cancel{5!}} = \frac{8\cdot7\cdot\cancel{6}}{\cancel{3}\cdot\cancel{2}\cdot1} = 56.$$

OR we could pick the 5 positions w/ 0's:

$$\binom{8}{5} = \frac{8!}{5!(8-5)!} = \frac{8!}{5!\cdot3!} = 56$$

Ex. How many 8-bit binary strings have <u>at least</u> 3 bits set to 1?

$$\frac{8!}{7!\,1!} = 8.$$

$$\frac{8!}{8!(8-8)!} = \frac{8!}{8!\cdot0!} = 1.$$

$$219 = \underline{\binom{8}{3} + \binom{8}{4} + \binom{8}{5} + \binom{8}{6} + \binom{8}{7} + \binom{8}{8}}$$

$$2^8 - \underline{\binom{8}{2} - \binom{8}{1} - \binom{8}{0}} = 219.$$