Summation

Very commonly, we want to add the terms of a sequence.
We use the notation:

$$\sum_{n=i}^{j} a_n = a_i + a_{i+1} + a_{i+2} + \dots + a_j .$$

n goes up to j

add these up

Greek capital sigma.

Start n at i

I prefer:

$$\sum_{i \le n \le j} a_n$$ — means the same thing.

eg.

$$\sum_{5 \le k \le 9} (k^2 - 3) = (5^2-3) + (6^2-3) + (7^2-3) + (8^2-3) + (9^2-3)$$
$$= \dots$$

eg. Write $1 + 3 + 5 + \dots + 99$ using Σ-notation.

$$\sum_{0 \le n \le 49} 2n+1$$

How to actually add this up? — we'll come back.

Algebraic rules for Σ-notation

(1) Sum of a constant

$$\sum_{a \le n \le b} 1 = b - a + 1.$$ (fenceposts!)

(2) factoring out a constant.

Constant — does not depend on n.

$$\sum_{n} k \cdot a_n = k \cdot a_i + k \cdot a_{i+1} + \dots + k \cdot a_j = k \sum_n a_n$$
$$= k(a_i + \dots + a_j)$$

eg.

$$\sum_{6\le n\le 13} 3 = 3\sum_{6\le n\le 13} 1 = 3(13-6+1) = 3\cdot 8 = 24.$$

③ Summing a sum

$$\sum_n (a_n + b_n) = \left(\sum_n a_n\right) + \left(\sum_n b_n\right)$$

because eg. $(a_0 + b_0) + (a_1 + b_1) + (a_2 + b_2) + \dots$

$$= (a_0 + a_1 + a_2 + \dots) + (b_0 + b_1 + b_2 + \dots)$$

④

$$\sum_{1\le k\le n} k = 1 + 2 + 3 + 4 + \dots + n$$ (nth triangular number)