Quantifiers

$\forall x : D.\ P(x)$ — $P(x)$ is true for every x in D

$\exists x : D.\ P(x)$ — $P(x)$ is true for at least one x in D.

eg.

① $\forall n : \mathbb{N}.\ \exists z : \mathbb{Z}.\ n < z$ True!

For every n, there will always be some z bigger than it.

② $\forall z : \mathbb{Z}.\ \exists n : \mathbb{N}.\ z < n$ True!

③ $\forall z : \mathbb{Z}.\ \exists n : \mathbb{N}.\ n < z$ False — eg. $z = -5$; no $\mathbb{N}$ is smaller.

④ $\exists n : \mathbb{N}.\ \forall z : \mathbb{Z}.\ n < z$ False.

⑤ $\exists z : \mathbb{Z}.\left(\forall n : \mathbb{N}.\ (z < n)\right)$ True — eg. $z = -13$.

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eg. ① "Every natural number is less than or equal to its own square."

$\forall n : \mathbb{N}.\ n \leq n^2$.

② "The square of any integer is nonnegative"

$\forall z : \mathbb{Z}.\ z^2 \geq 0$

③ "1369 is a perfect square"

$\exists s : \mathbb{Z}.\ s^2 = 1369$.

④ "n is even"

$\exists z : \mathbb{Z}.\ n/2 = z$ "n is divisible by 2"

$\boxed{\exists z : \mathbb{Z}.\ 2z = n.}$ "n is a multiple of 2".

Def'n. For an integer n,

$$\mathrm{Even}(n) \equiv \exists k:\mathbb{Z}.\ 2k = n.$$

(5) "n is odd"

$$\mathrm{Odd}(n) \equiv \neg\,\mathrm{Even}(n) \equiv \neg\left(\exists k:\mathbb{Z}.\ 2k = n\right)$$

$$\mathrm{Odd}(n) \equiv \exists k:\mathbb{Z}.\ n = 2k+1$$

(6) "If n is even, then n+2 is even."

$$\forall n:\mathbb{Z}.\ \mathrm{Even}(n) \rightarrow \mathrm{Even}(n+2)$$

De Morgan laws for quantifiers

$$\boxed{\neg\left(\forall x:D.\ P(x)\right) \equiv \exists x:D.\ \neg P(x)}$$

$$\neg\left(P(x_1) \wedge P(x_2) \wedge P(x_3) \wedge \cdots\right) \equiv \neg P(x_1) \vee \neg P(x_2) \vee \neg P(x_3) \vee \cdots$$

$$\boxed{\neg\left(\exists x:D.\ P(x)\right) \equiv \forall x:D.\ \neg P(x)}$$