Predicates

$x + 2 = 5$ — not a proposition on its own; depends on x.

We can define e.g.

$\underline{P(x)} = \underline{\text{"}x+2=5\text{"}}$ — predicate that takes x as input and results in a proposition.

e.g. $P(2)$ — false proposition $2+2=5$

$P(3)$ — true proposition $3+2=5$.

Universal Quantifier

Def'n If $P(x)$ is a predicate and D is some "domain" (i.e. collection of values, or a type), then

$$\forall x : D.\ P(x)$$

(pronounced "for all x in D, $P(x)$") is a proposition which is true iff $P(x)$ is true for every x in the domain D.

(Note: sometimes you see $\forall x \in D.\ P(x)$)

Ex.

- $\forall n : \mathbb{N}.\ P(n)$ $P(n) = \text{"}n+2=5\text{"}$ F
- $\forall n : \mathbb{N}.\ n \geq 0$ T
- $\forall x : \mathbb{Z}.\ x \geq 0$ F
- $\forall x : \mathbb{Z}.\ x^2 \geq 0$ T

Can be helpful to think of $\forall$ as a big AND:

$$\underline{\forall x : D.\ P(x)} \equiv P(x_1) \land P(x_2) \land P(x_3) \land \ldots$$

Ex. H = "Hendrix students who are over 11 feet tall"

$\forall h : H.$ h plays varsity pickleball.

Strange because H is empty! What should $\forall$ mean with an empty domain?

True!

- No counterexamples! So if it's not false... it should be true.
- If $\forall$ is like a big $\land$, what should you get if you $\land$ together zero things? <u>True</u>, because that is the identity element for $\land$.

<u>Existential Quantifier</u>

<u>Def'n</u> If $P(x)$ is a predicate and D a domain, then

$$\exists x : D.\ P(x)$$

("there exists an x in D such that $P(x)$") is a proposition which is true iff there is at least one x in D such that $P(x)$ is true.

Ex. $\exists n : \mathbb{N}.\ P(n)$ T – $P(3)$ is true

$\exists x : \mathbb{Z}.\ x^2 = 16$ T – eg. 4.

$\exists x : \mathbb{Z}.\ x^2 = 17$ F — no integers square to 17.

$\forall x : \mathbb{N}. \left(\exists y : \mathbb{N}. \left(x < y^2\right)\right)$