Propositional Logic

propositions $P, Q, \ldots$

NOT: $\neg P$
AND: $P \wedge Q$
OR: $P \vee Q$.

Implication

Defn Let P and Q be propositions. The conditional or implication $P \to Q$ "if P, then Q" is defined by the following truth table:

eg:
P = "You satisfy the requirements for an A"
Q = "You get an A."

P	Q	$P \to Q$
T	T	T
T	F	F
F	T	T
F	F	T

if $F = 0$ and $T = 1$, then $P \to Q$ is the same as $P \leq Q$.

P is called the premise or hypothesis
Q is called the conclusion.

Implication in English? Ways to say $P \to Q$:

- if P, then Q
- if P, Q
- Q if P (careful of order!!)
- P only if Q
- Q when P
- Q whenever P

Related conditionals:

Given $P \to Q$, we have:

- the <u>converse</u> is $Q \to P$
- the <u>inverse</u> is $\neg P \to \neg Q$
- the <u>contrapositive</u> is $\neg Q \to \neg P$

(the inverse of the converse or vice versa).

<u>Theorem</u>. An implication and its contrapositive always have the same truth value. On the other hand, an implication does <u>not</u> always have the same truth value as its converse or inverse.

P	Q	$P \to Q$	$\neg P$	$\neg Q$	$\neg Q \to \neg P$
T	T	T	F	F	T
T	F	F	F	T	F
F	T	T	T	F	T
F	F	T	T	T	T

<u>Def'n</u> Let P and Q be propositions. The <u>biconditional</u> $P \leftrightarrow Q$ (pronounced "P if and only if Q") is true when P and Q have the same truth values, and false otherwise. Alternatively, $P \leftrightarrow Q$ means $(P \to Q) \wedge (Q \to P)$.

P	Q	$P \leftrightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T

Note: often abbreviated "iff".

<u>Def'n</u> A proposition that is always true, no matter the value of the propositional variables that it contains, is a <u>tautology</u>. One that is always false is a <u>contradiction</u>.

e.g.

$$\left.\begin{array}{l} P \to P \\ P \lor \neg P \\ Q \to Q \end{array}\right\} \text{tautologies}$$

$$\left.\begin{array}{l} P \land \neg P \\ \neg(P \to P) \end{array}\right\} \text{contradiction}$$

<u>Def'n</u> Two propositions that always have the same truth value are <u>logically equivalent</u>, written $P \equiv Q$.

Alternatively, $P \equiv Q$ when $P \leftrightarrow Q$ is a tautology.

e.g. Show that $P \to Q \equiv \neg P \lor Q$.

We can show this w/ a truth table.

P	Q	$P \to Q$	$\neg P$	$\neg P \lor Q$
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

these columns are same, so logically equivalent.