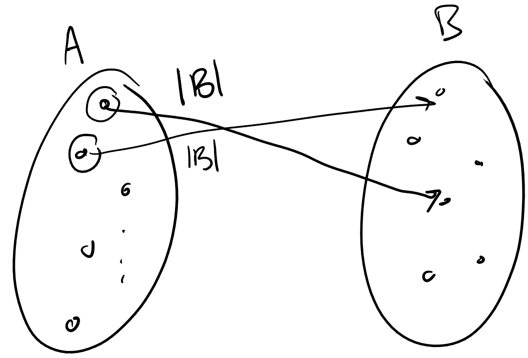
Given finite sets A, B, how many functions $A \to B$ are there?



For each element of A, we can independently choose which element of B to send it to.

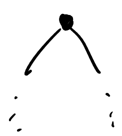
Total is $\underbrace{|B| \times |B| \times \dots \times |B|}_{|A|} = |B|^{|A|}$

Addition rule

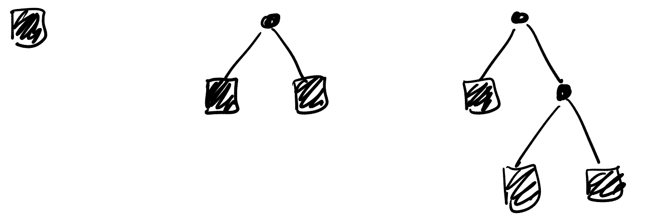
Suppose choosing something means either making one choice in c_1 ways, or making a different choice in c_2 ways, and the choices do not overlap at all. Then the total # of choices is $c_1 + c_2$.

eg. A binary tree is defined as either

- a leaf 
- OR a "branch" node with a left child + right child that are both binary trees.

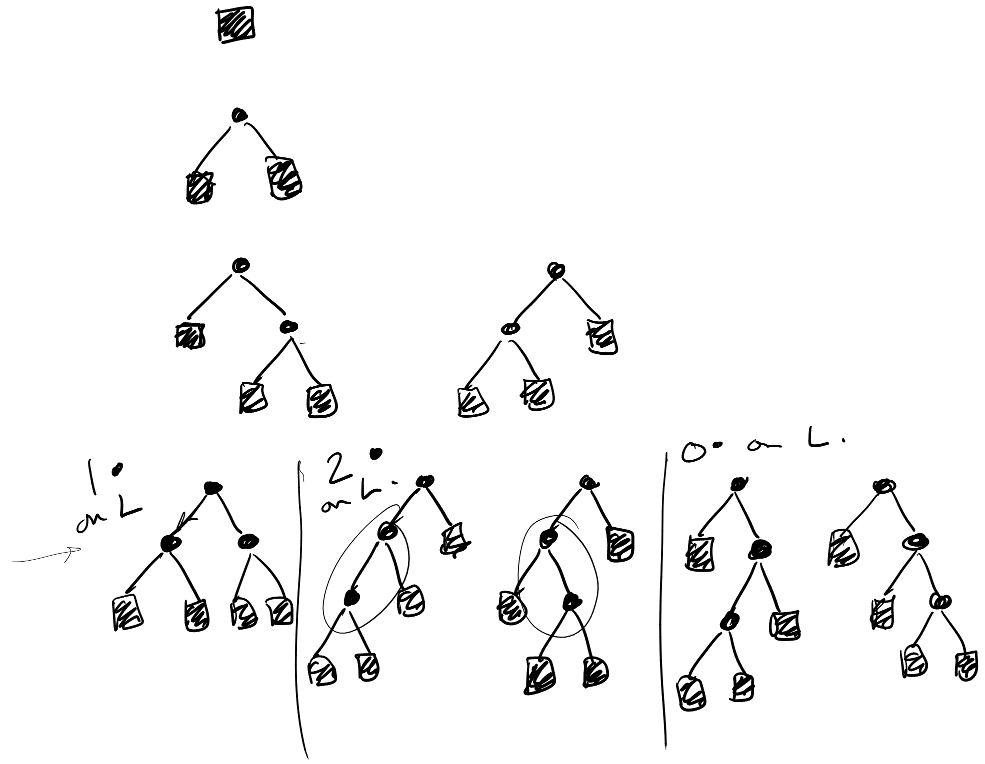


eg.



Q: how many different binary trees are there with n branch nodes? $= T(n)$

n	$T(n)$
0	1
1	1
2	2
3	5



In general,

$$T(0) = 1$$

$$T(n) = T(0)\cdot T(n-1) + T(1)\cdot T(n-2) + T(2)\cdot T(n-3) + \ldots + T(n-1)T(0).$$

$$T(1) = T(0)\cdot T(0) = 1\cdot 1 = 1.$$

$$T(2) = T(0)\cdot T(1) + T(1)\cdot T(0) = 1\cdot 1 + 1\cdot 1 = 2.$$

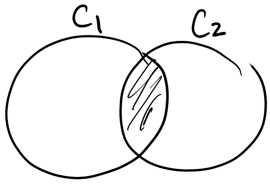
$$T(3) = T(0)\cdot T(2) + T(1)\cdot T(1) + T(2)\cdot T(0)$$
$$= 1\cdot 2 + 1\cdot 1 + 2\cdot 1 = 5$$

$$T(4) = T(0)T(3) + T(1)T(2) + T(2)T(1) + T(3)T(0)$$
$$= 1\cdot 5 + 1\cdot 2 + 2\cdot 1 + 5\cdot 1$$
$$= 14.$$

$$T(5) = 42$$

(Catalan numbers)

What if choices overlap?



If choosing something means making one choice in C_1 ways, OR another choice in C_2 ways, <u>and the choices overlap</u>, the total # of choices $= C_1 + C_2 -$ (# of choices in common).

(Principle of Inclusion - Exclusion, aka PIE)

eg. 2 restaurants. Restaurant A has 13 lunch options, Restaurant B has 10 lunch options, but 5 are identical. Your total # of lunch options is $13 + 10 - 5$.

eg. How many strings of 8 bits are there which either start w/ a 1 or end with 00? eg. 10110010, 00001000, 11100100

- How many 8-bit strings start w/ 1? 2^7.
- How many " " " end w/ 00? 2^6.
- How many " " " <u>both</u> start w/ 1 <u>and</u> end w/ 00? 2^5.

Therefore # of strings that start w/ 1 <u>or</u> end w/ 00 is

$$2^7 + 2^6 - 2^5 = 160.$$

eg. How many 5-letter strings start or end w/ a vowel? (AEIOUY)

- # that start w/ a vowel: $6 \cdot 26^4$
- # that end w/ a vowel: $6 \cdot 26^4$
- # that start <u>and</u> end w/ a vowel: $6 \cdot 6 \cdot 26^3$

So # that start <u>or</u> end w/ a vowel

$$= 6 \cdot 26^4 + 6 \cdot 26^4 - 6^2 \cdot 26^3.$$