Combinatorics = math of counting things

i.e. figuring out how big sets are.

- Basis for probability / statistics.
- Fundamental in analysis of algorithms.

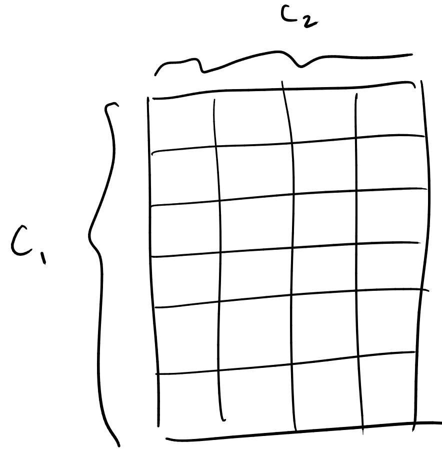
We will learn basic principles, not formulas.

The product rule

We will phrase things in terms of # of ways to choose an element of a set.

Suppose choosing an item can be broken down into two independent choices, with c_1 ways to make the first choice, and c_2 ways to make the second choice. Then the total # of ways to choose overall is the product $c_1 \cdot c_2$.

Independent = 2 choices do not affect each other.



i.e.

$$|A \times B| = |A| \times |B|$$

e.g. There are 3 kinds of cereal and 7 kinds of fruit. For breakfast you want to choose one of each. How many different breakfasts can you choose? $3 \times 7 = 21$.

e.g. NON-example: if some fruits are allowed w/ some cereals + not others, the choices are not independent.

e.g. How many 3-letter strings are there using A-Z?
e.g. AAA, HDX, CAR, ...

- 26×26 ways to choose 1st 2 letters
- $(26^2) \times 26$ ways to choose 2 letters + 1 letter.
- in general, 26^k ways to choose a k-letter string.

eg. How many subsets are there of an n-element set?

- We have an independent choice for each element: whether to include it in a subset or not.
- Hence, total # of ways to chose a subset B:

$$2 \times 2 \times 2 \cdots \times 2 = 2^n.$$

eg. Suppose we have 10 different cans of soup and we want to eat one per day. How many different ways could we do this?

- Choices are not independent! 10^{10} if eg. we went to the store each day and could get any kind we want.
- But after choosing one the first day, I can independently choose one of the 9 remaining on the 2nd day. One of 8 the next day, etc.
- So, total # ways $= 10 \cdot 9 \cdot 8 \cdot 7 \cdots 1 = 10!$

eg. What if we have 10 cans of soup but only want to eat soup for 4 days?

$$= 10 \cdot 9 \cdot 8 \cdot 7 .$$

$$= \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}} = \frac{10!}{6!}$$

In general, $\dfrac{n!}{(n-k)!}$ ways to eat from n soups for k days.

eg. Given finite sets A and B, how many different functions are there $A \to B$?