Solving modular equivalences

eg. Solve for x:

$$x + 7 \equiv_3 12$$

- Since $\equiv_3$ is reflexive, $-7 \equiv_3 -7$.
- since $\equiv_3$ is a congruence w.r.t. addition,

$$x + 7 + (-7) \equiv_3 12 + (-7).$$

$$\rightarrow \quad x \equiv_3 5.$$

- Now, since $5 \equiv_3 2$ and $\equiv_3$ is transitive,

$$x \equiv_3 2.$$

Note this means $x \in \{\ldots, -4, -1, 2, 5, 8, 11, \ldots\}$.

eg. $x + 28 \equiv_5 12$

$\rightarrow$ $x \equiv_5 -16$

$\rightarrow$ $x \equiv_5 -1$ or $x \equiv_5 4$

eg. $101x + 52 \equiv_{10} 68$

$\rightarrow$ $101x \equiv_{10} 16 \equiv_{10} 6$.

(Can't divide by 101 !)

Note: since $101 \equiv_{10} 1$, therefore $101 \cdot x \equiv_{10} 1 \cdot x$

(because $\equiv_{10}$ is a congruence w.r.t. multiplication).

$\rightarrow$ $x \equiv_{10} 6.$

$$3x + 19 \equiv_7 2(x - 73)$$

$$3x + 19 \equiv_7 2x - 146$$

$\rightarrow 3x + 5 \equiv_7 2x - 6$

$\rightarrow 3x \equiv_7 2x - 11$

$\rightarrow 3x \equiv_7 2x - 4$

$\rightarrow x \equiv_7 -4$

$$x + 22 \equiv_{19} 20x + 3$$

$\rightarrow x + 19 \equiv_{19} 20x$

$\rightarrow x \equiv_{19} 20x$

$\rightarrow 0 \equiv_{19} \underline{19}x$

$\rightarrow 0 \equiv_{19} 0.$ Always true for any x.

$$x + 22 \equiv_{19} 20x - 7$$

$\rightarrow x + 29 \equiv_{19} 20x$

$\rightarrow x + 10 \equiv_{19} 20x$

$\rightarrow 10 \equiv_{19} 19x \equiv_{19} 0.$ No solutions.

$3x \equiv_7 5$ — don't know how to solve (yet!)