Summation.

Capital Greek sigma

$$\sum_{n=i}^{j} a_n = a_i + a_{(i+1)} + a_{i+2} + \dots + a_j$$

eg. $$\sum_{k=2}^{6} (3k-1) = (3\cdot 2 - 1) + (3\cdot 3 - 1) + \dots + (3\cdot 6 - 1)$$

Alternatively: $$\sum_{i \le n \le j} a_n = a_i + \dots + a_j \quad \text{(same thing)}$$

eg. Write $(1+3+5+7+\dots+99)$ using Σ-notation.

$$\sum_{0 \le n \le 49} (2n+1)$$

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① $$\sum_{a \le n \le b} 1 = \underbrace{1+1+1+\dots+1}_{b-a+1}.$$

② k is a constant, ie. doesn't depend on n.

$$\sum_n k \cdot a_n = k\cdot a_0 + k\cdot a_1 + k \cdot a_2 + \dots$$
$$= k(a_0 + a_1 + \dots)$$
$$= k \sum_n a_n .$$

eg. $$\sum_{2 \le n \le 7} 6 = 6 \sum_{2 \le n \le 7} 1 = 6(7-2+1) = 36.$$

③ $$\sum_n (a_n + b_n) = \left(\sum_n a_n\right) + \left(\sum_n b_n\right)$$

$a_0 + b_0 + a_1 + b_1 + a \dots$ $\qquad$ $(a_0 + a_1 + \dots) + (b_0 + b_1 + \dots)$

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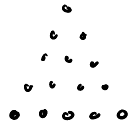
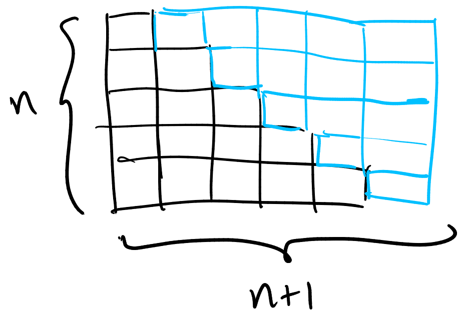
$$\sum_{1 \le k \le n} k = 1 + 2 + 3 + \dots + n = \Delta_n$$



$$\begin{aligned}
\Delta_n &= 1 + 2 + 3 + \dots + (n-1) + n \\
+ \quad \Delta_n &= n + (n-1) + (n-2) + \dots + 2 + 1 \\
\hline
2\Delta_n &= (n+1) + (n+1) + (n+1) + \dots + (n+1) + (n+1) \\
&= n(n+1)
\end{aligned}$$

$$\Rightarrow \Delta_n = \frac{n(n+1)}{2}$$



half the area $= \dfrac{n(n+1)}{2}$.

$$\begin{aligned}
\sum_{0 \le n \le 49} (2n+1) &= \left(\sum_{0 \le n \le 49} 2n\right) + \left(\sum_{0 \le n \le 49} 1\right) \\
&= 2\left(\sum_{0 \le n \le 49} n\right) + \left(\sum_{0 \le n \le 49} 1\right) \\
&= 2\left(\sum_{0 \le n \le 49} n\right) + 50 \\
&= 2\left(\sum_{1 \le n \le 49} n\right) + 50 \\
&= 2\left(\frac{49 \cdot 50}{2}\right) + 50 \\
&= 49 \cdot 50 + 50 \\
&= 50(49+1) = 50 \cdot 50 \\
&= 2500.
\end{aligned}$$

$$\sum_{i \le n \le j} a_n = a_i + \sum_{i+1 \le n \le j} a_n$$

S

$$1 + r + r^2 + r^3 + \dots + r^n = S$$

i.e. $\displaystyle\sum_{0 \le k \le n} r^k$.

$$
\begin{aligned}
S &= 1 + r + r^2 + r^3 + \dots + r^n \\
-\ r \cdot S &= \phantom{1 +{}} r + r^2 + r^3 + \dots + r^n + r^{n+1} \\
\hline
S - rS &= 1 + 0 + 0 + \dots - r^{n+1}
\end{aligned}
$$

$$S(1-r) = 1 - r^{n+1}$$

$$S = \frac{1 - r^{n+1}}{1-r}$$

e.g.

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \frac{1}{256}$$

$$= \sum_{0 \le k \le 8} \left(\frac{1}{2}\right)^k = \frac{1 - \left(\frac{1}{2}\right)^9}{1 - \frac{1}{2}} = \dots = 2 - \frac{1}{256}.$$

magic
(calculator)