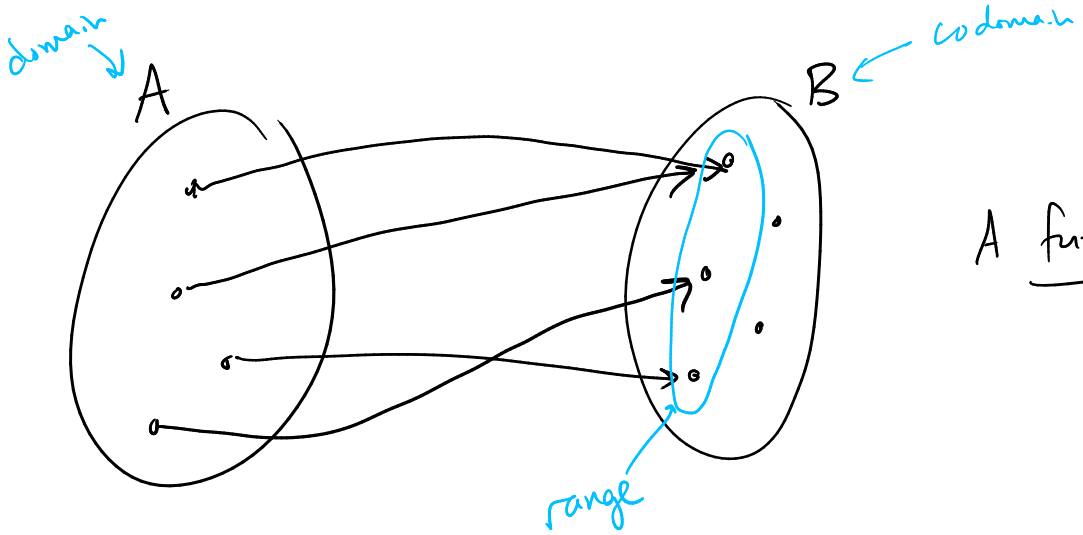
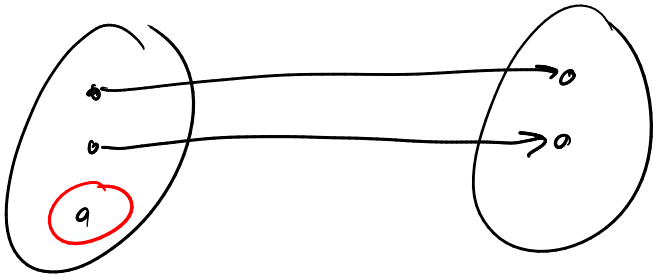
Functions

Defn Let A and B be sets. A **function** f from A to B, written $f: A \to B$, is a relation from A to B such that every $a \in A$ is related to exactly one $b \in B$.

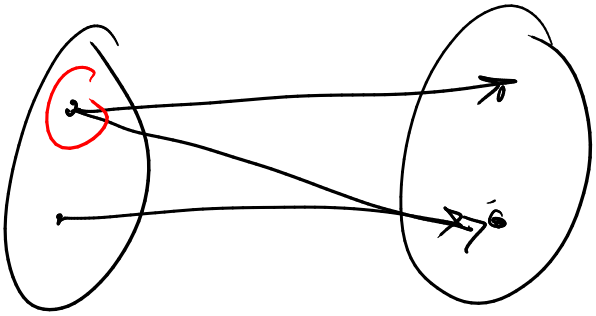
A is called the **domain** of f, and B is called the **codomain**. We write $f(a) = b$ to denote the unique b related to a given a.



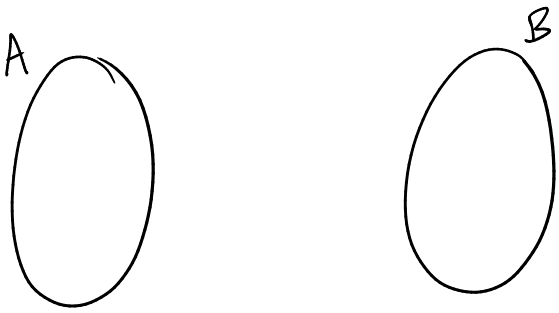
A function!



Not a function.
(partial function)



Not a function.



✓ empty function.

Defn The range of a function $f: A \to B$ is the set

$$\{ b \mid b \in B, \ \exists a : A . \ f(a) = b \}$$

ie all $b \in B$ with at least one "incoming arrow".

eg. Let $Shp = \{ \diamond, \triangle, \square \}$. Define $g : Shp \to \mathbb{N}$ by

$$g(\diamond) = 6$$
$$g(\triangle) = 6$$
$$g(\square) = 32.$$

eg. Let $h : \mathbb{N} \to \mathbb{N}$ be defined by $h(n) = 2n$.

range = all even natural #'s.

eg. Let $\ell : \mathbb{N} \to Shp$ be the function which sends all multiples of 3 to $\diamond$, all numbers 1 more than a multiple of 3 to $\square$, all 2 more than multiple of 3 to $\triangle$. ie.

$$\ell(3n) = \diamond$$
$$\ell(3n+1) = \square$$
$$\ell(3n+2) = \triangle.$$