Set operations

Def'n The cardinality of a finite set is the number of distinct elements.

Cardinality of S is written $|S|$.

e.g. $|\{1,2,3\}| = 3$

$|\emptyset| = 0$.

$|\{1,3,\ldots,99\}| = 50$

Cartesian product

If A and B are sets, the Cartesian product of A and B, written $A \times B$, is the set of all possible ordered pairs of elements from A and B.

$$A \times B = \{(a,b) \mid a \in A,\ b \in B\}.$$

Ex. $A = \{1,2,3\}$, $B = \{\Delta, \square\}$.

Then $A \times B = \{(1,\Delta), (2,\Delta), (3,\Delta), (1,\square), (2,\square), (3,\square)\}$.

		B: Δ	$\square$
A	1	$(1,\Delta)$	$(1,\square)$
	2	$(2,\Delta)$	$(2,\square)$
	3	$(3,\Delta)$	$(3,\square)$

$B \times A = \{(\Delta,1), (\Delta,2), \ldots$

So in general, $|A \times B| = |A| \times |B|$ (for finite sets).

e.g. $\mathbb{N} \times \mathbb{Q} = \{(3, 1/3), (7, 49/18), (6, -212/3), \ldots\}$

Note order matters! $A \times B \neq B \times A$.

Power set

List all subsets of $\{1,2,3\}$.

Def'n The power set of a set A, written $\mathcal{P}(A)$, is the set of all subsets of A.

e.g. $\mathcal{P}(\{1,2,3\}) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$

1?	2?	3?	subset
T	T	T	$\{1,2,3\}$
T	T	F	$\{1,2\}$
T	F	T	$\{1,3\}$
T	F	F	$\{1\}$
F	T	T	$\{2,3\}$
F	T	F	$\{2\}$
F	F	T	$\{3\}$
F	F	F	$\emptyset$

Each element can be independently in or out of the set, so each new element doubles the # of possible subsets.

So $|\mathcal{P}(A)| = 2^{|A|}$.

e.g. $A = \{\text{dog}\}$, $\mathcal{P}(A) = \{\emptyset, \{\text{dog}\}\}$

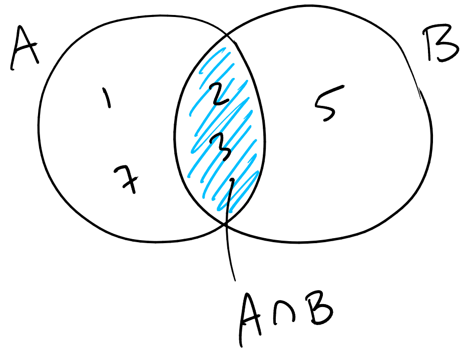
$|A| = 1$. $2^1 = 2$.

e.g. $A = \emptyset$, $\mathcal{P}(A) = \{\emptyset\}$

$|A| = 0$ $|\mathcal{P}(A)| = 1$ $2^0 = 1$. ✓

Def'n The intersection of sets A and B is the set of elements common to both:

$$A \cap B = \{x \mid x \in A \land x \in B\}.$$



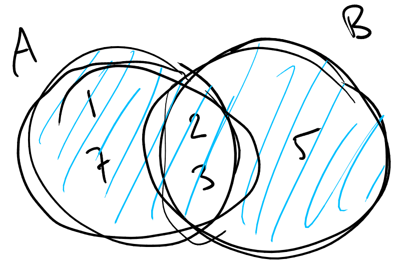
e.g. $A = \{1, 2, 7, 3\}$

$B = \{2, 3, 5\}$

$A \cap B = \{2, 3\}$.

Def'n The union of sets A and B is the set of elements contained in either:

$$A \cup B = \{x \mid x \in A \lor x \in B\}.$$



e.g. $A \cup B = \{1, 7, 2, 3, 5\}$.

$$|A \cup B| \leq |A| + |B|$$

$$|A \cup B| = |A| + |B| - |A \cap B|$$

(principle of inclusion-exclusion (PIE))

$$|A \cup B| + |A \cap B| = |A| + |B|.$$