Set Theory

Def'n A *set* is an unordered, finite or infinite collection of objects, called "elements" or "members" of the set. Sets cannot contain a given element more than once, and the order of the elements in a set does not matter.

Put another way, the only thing that matters about a particular element is whether it is in a set or not.

We typically use capital letters for sets and lowercase for elements.

Def'n The notation $x \in S$ is a proposition, pronounced "x is an element of S", or "x is in S".

Examples / ways to write sets

$\{1, 3, 5, 7, 9\}$ — a literal set in curly braces with 5 element.

$\{1, 2, 3, \ldots, 100\}$ — set containing all integers from 1 up to 100.

$\{1, 3, 5, \ldots, 99\}$ — all odd #'s 1 up to 99.

$\{\text{cow}, \text{horse}, \text{dog}\}$ — set w/ 3 elements.

$\{53\}$ — set w/ one element.

$\{\}$ — empty set, w/ 0 elements.

also written $\emptyset$.

We can also write sets using set builder notation:

- $\{ x \mid x \in \mathbb{Z},\ 0 < x \leq 5 \}$

  elements. "such that" conditions.

  $= \{1, 2, 3, 4, 5\}$

$\{\{1,2\}, \{3\}, \{1,3\}\}$

- $\{ x \mid x \in \mathbb{Z},\ \text{Odd}(x),\ -13 \leq x \leq 100 \}$

  $= \{-13, -11, -9, \ldots, 97, 99\}$

- $\{ 2x+5 \mid x \in \mathbb{N},\ x \leq 10 \}$

  $= \{5, 7, 9, \ldots, 25\}$.

- $\{ (x,y) \mid x \in \mathbb{Z},\ y \in \mathbb{Z},\ x+y = 6 \}$

  $= \{(2,4), (-20, 26), \ldots \}$

Some special sets

- $\mathbb{N}$ = natural #'s = $\{0, 1, 2, 3, \dots\}$
- $\mathbb{Z}$ = integers = $\{\dots, -2, -1, 0, 1, 2, \dots\}$

  Zahlen = German for "numbers"
- $\mathbb{Q}$ = rationals = $\{\boxed{p/q} \mid p \in \mathbb{Z}, q \in \mathbb{Z}, q \neq 0\}$.

  $\left(\frac{a}{b} = \frac{c}{d}\right)$ when $a \cdot d = b \cdot c$.
- $\mathbb{R}$ = real #'s.
- $\mathbb{C}$ = Complex #'s.