Algebraic laws of Propositions.

p	q	$p \wedge q$				
T	T	T	swap $\rightarrow$	F	F	F
T	F	F		F	T	T
F	T	F		T	F	T
F	F	F		T	T	T

Organizing principles:

1. Most laws have an "opposite world" equivalent: Swap $\wedge / \vee$ and T/F, law remains true.
2. We can get some intuition by thinking in terms of $+ / \times$:
   - F is like 0
   - T is like 1
   - $\wedge$ is like multiplication
   - $\vee$ is kind of like $+$, but w/ a max value of 1.

Name	Law	Opposite world law
Identity	$p \wedge T \equiv p$	$p \vee F \equiv p$
Annihilation	$p \wedge F \equiv F$	$p \vee T \equiv T$.
Idempotence	$p \wedge p \equiv p$	$p \vee p \equiv p$.
Commutativity	$p \wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$
Associativity	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	$(p \vee q) \vee r \equiv p \vee (q \vee r)$.
Distributivity	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
De Morgan	$\neg(p \wedge q) \equiv \neg p \vee \neg q$	$\neg(p \vee q) \equiv \neg p \wedge \neg q$.
Contradiction / Excluded Middle	$p \wedge \neg p \equiv F$	$p \vee \neg p \equiv T$

Double Negation	$\neg(\neg p) \equiv p$
Implication	$p \rightarrow q \equiv \neg p \vee q$
Iff	$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

Example Show that $(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$.

relationship to next line

$(p \rightarrow q) \wedge (p \rightarrow r)$

$\equiv$ { Implication } reason relationship is true

$(\neg p \vee q) \wedge (\neg p \vee r)$

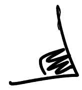
$\equiv$ { Distributivity }

$\neg p \vee (q \wedge r)$

$x \cdot y + x \cdot z$
$= x \cdot (y + z)$

$\equiv$ {Implication}

$p \to (q \wedge r)$



While ($\underbrace{\text{tickets} < 30}_{T}$ or (! $\underbrace{\text{heffalump.isEvil}}_{E}$ () and not ($\underbrace{\text{count} > 5}_{C}$)))

$$T \vee (\neg E \wedge \neg C)$$

stops when this is false. ie

$$\neg(T \vee (\neg E \wedge \neg C))$$

$\equiv$ {De Morgan}

$$\neg T \wedge \neg(\neg E \wedge \neg C)$$

$\equiv$ { " " }

$$\neg T \wedge (\underline{\neg(\neg E)} \vee \underline{\neg(\neg C)})$$

$\equiv$ {Double negation}

$$\neg T \wedge (E \vee C)$$