Implication

Defn If p, q are propositions, the **conditional** or **implication** "if p then q" is written $p \to q$ and pronounced "p implies q"
("if p then q", "q if p", ...)
"q when p"

premise/hypothesis

conclusion.

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Defn Given an implication $p \to q$,

- the **converse** is $q \to p$
- the **inverse** is $\neg p \to \neg q$
- the **contrapositive** is $\neg q \to \neg p$.

Thm An implication and its contrapositive always have the same truth value. On the other hand, the converse + inverse are not necessarily the same.

(Proof: make a truth table!)

Defn Let p, q be propositions. The **biconditional** $p \leftrightarrow q$ ("p if and only if q", "p iff q") is true when p and q have the same truth value, and false otherwise.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

eg. $(p \to q) \leftrightarrow (\neg q \to \neg p)$ is always true, no matter what p, q are.

Defn A proposition that is always true is called a **tautology**.

eg. $p \vee \neg p$

$p \to p$

$p \leftrightarrow p$

$p \vee T$

Defn Two propositions that always have the same truth value are **logically equivalent**, written $p \equiv q$. Alternatively, $p \equiv q$ when $(p \leftrightarrow q)$ is a tautology.

eg. Show that $p \to q \equiv \neg p \vee q$.

Make a truth table!

p	q	$p \to q$	$\neg p \vee q$
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

same!