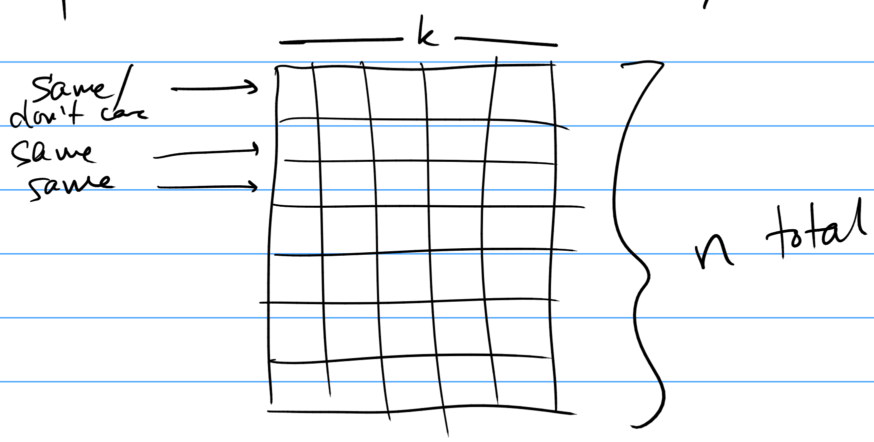
10 soups – 4 days: $10 \times 9 \times 8 \times 7 = \frac{10!}{6!}$

In general: n soups – k days:

$$n \times (n-1) \times (n-2) \times \dots \times (n-k+1) = \frac{n!}{(n-k)!}$$

Question: what if I don't care what order I eat the soup? ie. how many different subsets of 4 soups are there out of the set of 10?

Division rule Suppose we have n total choices, but we want to think of some of them as equivalent (ie. we don't care about the difference between some of them). If the choices always come in groups of exactly k choices which are all equivalent, then the number of "really different" choices is n/k.



Ex. (a) How many ways are there to seat 5 people at a table? $5! = 5 \times 4 \times 3 \times 2 \times 1$, – 5 choices for 1st seat, 4 choices for next, etc.

(b) The table is circular, and we don't care who is in which seat, only about who is sitting to the left + right of who.

$5!$ is the total # of ways to seat them, but every seating arrangement is equivalent to 4 others (rotations).

So, by division rule, # of different arrangements is
$5!/5 = 4!$.

(c) We also don't care about left/right, just who is sitting next to who.

Each configuration is equivalent to its mirror image, so $4!/2 = 12$.

<u>Ex.</u> Choosing 4 soups out of 10.

We know there are $10 \times 9 \times 8 \times 7 = \frac{10!}{6!}$ schedules for 4 soups. Each schedule is equivalent to <u>4!</u> others (# of ways to order 4 particular soups).
So by the division rule, # of <u>subsets</u> of 4 soups is

$$\frac{10 \times \cancel{9}^{3} \times \cancel{8} \times 7}{\cancel{4} \times \cancel{3} \times \cancel{2} \times 1} = 210. = \frac{10!}{6!\,4!}$$

In general, if we have a set of size n, the number of ways to choose a subset of k things (when we don't care about order) is

$$\frac{n!}{(n-k)!\,k!} = \binom{n}{k}$$

This is called a <u>binomial coefficient</u>, and pronounced "n choose k".

$n-(n-k) = n-n+k = k.$

$\binom{7}{2} = \frac{7\cdot 6}{2\cdot 1} = 21$

triangular #'s

n \ k		0	1	2	3	4	5	6	7	
0		1								1
1	1	1	1							2
2	1	1	2	1						4
3	2	1	3	3	1					8
4	3	1	4	6	4	1				16
5	5	1	5	10	10	5	1			
6	8	1	6	15	20	15	6	1		
7	13	1	7	21	35	35	21	7	1	

sum of row $n = 2^n$

Sums of diagonals are Fibonacci #'s.

(Pascal's Triangle)

$$\binom{n}{k} = \frac{n!}{(n-k)!\,k!}$$

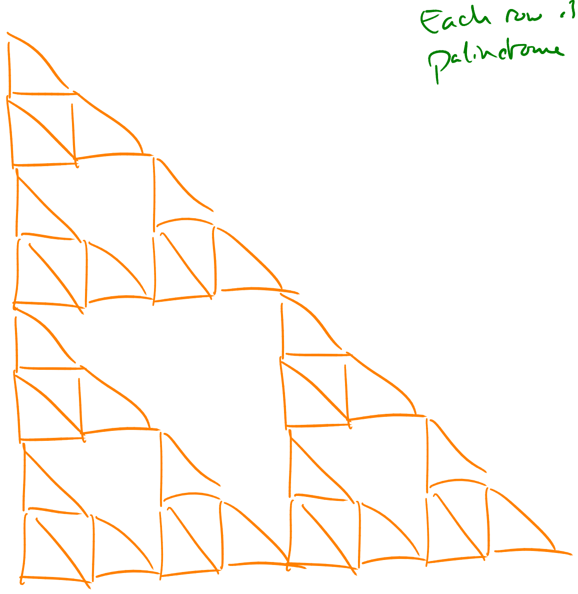
$$\binom{6}{2} = \frac{6!}{4!\,2!} = \frac{6\cdot 5}{2\cdot 1} = 3\cdot 5 = 15$$

$$\binom{n}{n} = 1$$

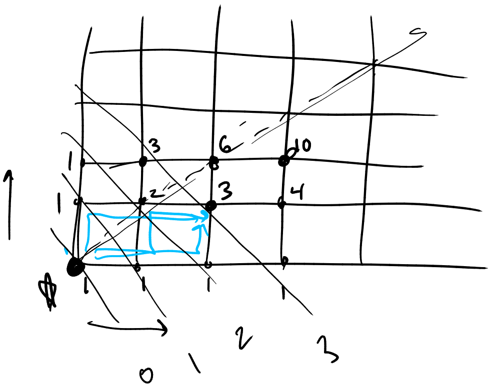
$$\binom{n}{n-1} = n = \binom{n}{1}$$

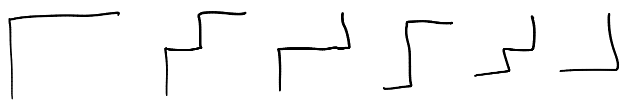
Each is sum of # above + above-left → $\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$

Each row is palindrome $\binom{n}{k} = \binom{n}{n-k}$



# of ways to walk from (0,0) to (x,y) only going N or E $= \binom{x+y}{x} = \binom{x+y}{y}$





Pascal's triangle again!

Choosing which x steps out of x+y total should go E.