$\forall$ / all, each

1. "Every $\mathbb{N}$ is $\leq$ its own square".

$$\forall n : \mathbb{N}.\ n \leq n^2$$

2. "1369 is a perfect square".

a) Let $Sq(n)$ be the proposition that n is a perfect square. $Sq(1369)$. — Or depending on desired level of detail!

b) $\exists n : \mathbb{N}.\ n^2 = 1369.$ ✓

alternatively, "1369's square root is a natural number".

$\sqrt{1369} : \mathbb{N}$? ← ": $\mathbb{N}$" is not a question we can ask.

What does $\sqrt{\ }$ mean? → Finding the n whose square is 1369.

Nested quantifiers

$(\forall \ldots (\exists \ldots (\forall \ldots (\exists \ldots \text{blah}))))$

① $\forall n : \mathbb{N}. (\exists z . \mathbb{Z}.\ n < z)$ ✓

② $\forall z : \mathbb{Z}.\ \exists n : \mathbb{N}.\ z < n$ ✓

③ $\forall z : \mathbb{Z}.\ \exists n : \mathbb{N}.\ n < z$ ✗

④ $\exists n : \mathbb{N}.\ \forall z : \mathbb{Z}.\ n < z$ ✗

⑤ $\exists z : \mathbb{Z}.\ \forall n : \mathbb{N}.\ z < n$ ✓

Example of negating nested quantifiers:

$$\neg (\forall n : \mathbb{N}.\ \exists z : \mathbb{Z}.\ n < z)$$

$$\equiv$$

$$\exists n : \mathbb{N}.\ \neg (\exists z : \mathbb{Z}.\ n < z)$$

$$\equiv$$

$$\exists n : \mathbb{N}.\ \forall z : \mathbb{Z}.\ \neg (n < z) \equiv \exists n : \mathbb{N}.\ \forall z : \mathbb{Z}.\ n \geq z.$$

More examples / restricted quantifiers.

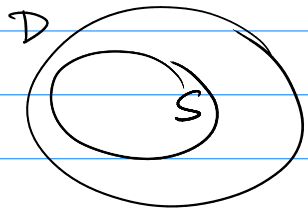
ex. "Every even number is a perfect square." (false).

$$\forall n : \mathrm{Even} .\ \mathrm{Sq}(n).$$

$\equiv$ OK, but goal: translate to equivalent proposition with a simpler domain.

$$\forall n : \mathbb{N}.\ \mathrm{Even}(n) \to \mathrm{Sq}(n).$$

In general, suppose D is some domain and S is a subset of D.



Let $S(x)$ be the predicate meaning: x is in the domain S.

$$\forall x : S.\ P(x) \equiv \forall x : D.\ S(x) \to P(x). \leftarrow$$

Aside: how is $\mathrm{Even}(n)$ defined?

$$\mathrm{Even}(n) = \exists k : \mathbb{Z}.\ 2k = n.$$