Predicates & Quantifiers.

Goal: talk formally about logical statements involving "all" or "some".

<u>Def'n</u> If $P(x)$ is a predicate (i.e. a function that returns a Proposition) and D is some "domain" (i.e. a set of values, i.e. a type), then

$$\forall x : D.\ P(x)$$

(pronounced "for all x in D, $P(x)$") is a proposition which is true iff $P(x)$ is true for <u>all</u> x in the domain D.

(People often also use notation $\forall x \in D.\ P(x)$)
(D called "domain of discourse"; sometimes omitted)

Note, we can think of $\forall$ as being like a giant "and". That is,

$$\forall x : D.\ P(x) \approx P(d_1) \wedge P(d_2) \wedge P(d_3) \wedge \ldots$$

where $d_1, d_2, d_3, \ldots$ are all the values in domain D.
This helps us understand properties of $\forall$.

eg:
$$\begin{aligned} & \neg(\forall x : D.\ P(x)) \\ \approx\ & \neg(P(d_1) \wedge P(d_2) \wedge P(d_3) \wedge \ldots) \\ \equiv\ & \neg P(d_1) \vee \neg P(d_2) \vee \neg P(d_3) \vee \ldots \\ \equiv\ & \exists x : D.\ \neg P(x). \end{aligned}$$

__Defn.__ If $P(x)$ is a predicate and D is a domain, then

$$\exists x : D.\ P(x)$$

(pronounced "there exists x in D such that $P(x)$") is a proposition which is true iff there is at least one value of x in domain D for which $P(x)$ is true.

kind of like $\exists x : D.\ P(x) \approx P(d_1) \vee P(d_2) \vee P(d_3) \vee \ldots$

Hence

$$\neg(\forall x : D.\ P(x)) \equiv \exists x : D.\ \neg P(x)$$

$$\neg(\exists x : D.\ P(x)) \equiv \forall x : D.\ \neg P(x).$$