Discrete Math HW 4: Learning goals S1, S2 (solutions)

S1: I can state the definitions, and determine membership, of standard sets such as $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, and $\mathbb{R}$.

Exercise 1 Name three examples of numbers which are members of $\mathbb{Z}$ but not $\mathbb{N}$.

$\mathbb{N}$ includes zero and all positive whole numbers; $\mathbb{Z}$ includes negative numbers as well. Therefore, -3, -7, and -49 are three examples of numbers in $\mathbb{Z}$ but not $\mathbb{N}$.

Exercise 2 Name three examples of numbers which are in $\mathbb{Q}$ but not $\mathbb{Z}$.

$\mathbb{Q}$ includes all positive and negative fractions. Three examples of elements of $\mathbb{Q}$ which are not in $\mathbb{Z}$ are $1/2$, $19/47$, and $-2/3$.

Exercise 3 For the purposes of this class, is $0 \in \mathbb{N}$?

Yes, 0 is an element of the set of natural numbers $\mathbb{N}$.

Exercise 4 Which of $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, and $\mathbb{R}$ are closed under

- addition? *All these sets are closed under addition.*

- subtraction? *All but $\mathbb{N}$ are closed under subtraction.*

- multiplication? *All these sets are closed under multiplication.*

- division? *$\mathbb{Q}$ and $\mathbb{R}$ are closed under division (as long as you do not divide by zero).*

S2: I can evaluate and construct sets using union, intersection, difference, and complement of sets, and sets defined via set builder notation.

Exercise 5 List the elements of each of the following sets.

1. $\{n \mid n \in \mathbb{N}, n < 6\} = \{0, 1, 2, 3, 4, 5\}$

2. $\{n \mid n \in \mathbb{Z}, n < 6, n > -2\} = \{-1, 0, 1, 2, 3, 4, 5\}$

3. $\{n^2 + 3 \mid n \in \mathbb{Z}, -2 \le n \le 3\} = \{3, 4, 7, 12\}$

4. $\{(a, b) \mid a \in \mathbb{Z}, b \in \mathbb{N}, b \le 9, a^2 = b\} = \{(0,0), (1,1), (-1,1), (2,4), (-2,4), (3,9), (-3,9)\}$

5. $\{xy \mid x \in \mathbb{N}, y \in \mathbb{N}, x \le 4, y \le 4\} = \{0, 1, 2, 3, 4, 6, 8, 9, 12, 16\}$

Exercise 6 Complete the Disco exercises in `s2a.disco`, which you can find linked from the course website.