

Discrete Math HW 8: Learning goals R1, R2

due Wednesday, April 1

R1: I can compute the terms of a sequence defined via a recurrence, and implement a recurrence as a recursive Disco function.

Exercise 1 For each sequence defined below, (1) write out the first five terms of the sequence by hand; (2) define the sequence as a Disco function, and check that the first five terms agree with what you wrote.

Note: when multiplying the output of a function by a constant in Disco, you will have to write a multiplication sign, e.g. $3 * c(n - 1)$ instead of $3c(n - 1)$.

(a)

$$\begin{aligned}a_0 &= 1 \\ a_n &= a_{n-1} + 3 \quad (n \geq 1)\end{aligned}$$

(b)

$$\begin{aligned}b_0 &= 7 \\ b_n &= b_{n-1} - 1 \quad (n \geq 1)\end{aligned}$$

(c)

$$\begin{aligned}c_0 &= 0 \\ c_n &= 3c_{n-1} - 1 \quad (n \geq 1)\end{aligned}$$

(d)

$$\begin{aligned}d_0 &= 0 \\ d_n &= d_{n-1} + n^2 \quad (n \geq 1)\end{aligned}$$

(e)

$$\begin{aligned}e_0 &= 1 \\ e_1 &= 1 \\ e_n &= 2e_{n-1} + e_{n-2} \quad (n \geq 2)\end{aligned}$$

R2: I can evaluate sums involving arithmetic and geometric series.

Exercise 2 Evaluate each of the following sums. Show your work.

For credit, complete at least 4.

If you want, you can use Disco to check your results. For example:

```
sum : List(Z) -> Z
sum(zs) = reduce(~+~, 0, zs)
```

```
Disco> sum [2i - 3 | i in [0 .. 10]]
```

(a)

$$\sum_{0 \leq i \leq 10} (2i - 3)$$

(b)

$$\sum_{1 \leq k \leq 20} (5 - k)$$

(c)

$$\sum_{0 \leq j \leq 50} (3j - 7)$$

(d)

$$7 + 10 + 13 + 16 + 19 + \cdots + 100$$

(e)

$$\sum_{7 \leq i \leq 29} i$$

(f)

$$\sum_{0 \leq k \leq 10} (2^k - 1)$$

(g)

$$1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \cdots + \frac{1}{3^7}$$

