Discrete Math HW 8: Learning goals R1, R2 (solutions)

due Wednesday, April 1

R1: I can compute the terms of a sequence defined via a recurrence, and implement a recurrence as a recursive Disco function.

Exercise 1 For each sequence defined below, (1) write out the first five terms of the sequence by hand; (2) define the sequence as a Disco function, and check that the first five terms agree with what you wrote.

(a)

$$a_0 = 1$$
$$a_n = a_{n-1} + 3 \quad (n \geq 1)$$

$$1, 4, 7, 10, 13, \ldots$$

```
!!! each(a, [0..4]) == [1, 4, 7, 10, 13]
a : N -> N
a(0) = 1
a(n) = a(n.-1) + 3
```

(b)

$$b_0 = 7$$
$$b_n = b_{n-1} - 1 \quad (n \geq 1)$$

$$7, 6, 5, 4, 3, \ldots$$

```
!!! each(b, [0..4]) == [7, 6, 5, 4, 3]
b : N -> Z
b(0) = 7
b(n) = b(n .- 1) - 1
```

(c)

$$c_0 = 0$$
$$c_n = 3c_{n-1} - 1 \quad (n \geq 1)$$

$$0, -1, -4, -13, -40, \ldots$$

```
!!! each(c, [0..4]) == [0, -1, -4, -13, -40]
c : N -> Z
c(0) = 0
c(n) = 3*c(n.-1) - 1
```

(d)

$$\begin{aligned} d_0 &= 0 \\ d_n &= d_{n-1} + n^2 \quad (n \geq 1) \end{aligned}$$

$$0, 1, 5, 14, 30, \ldots$$

```
!!! each(d, [0..4]) == [0, 1, 5, 14, 30]
d : N -> N
d(0) = 0
d(n) = d(n .- 1) + n^2
```

(e)

$$\begin{aligned} e_0 &= 1 \\ e_1 &= 1 \\ e_n &= 2e_{n-1} + e_{n-2} \quad (n \geq 2) \end{aligned}$$

$$1, 1, 3, 7, 17 \ldots$$

```
!!! each(e, [0..4]) == [1, 1, 3, 7, 17]
e : N -> N
e(0) = 1
e(1) = 1
e(n) = 2 * e(n .- 1) + e(n .- 2)
```

R2: I can evaluate sums involving arithmetic and geometric series.

Exercise 2 Evaluate each of the following sums. Show your work.
For credit, complete at least 4.

If you want, you can use Disco to check your results. For example:

```
sum : List(Z) -> Z
sum(zs) = reduce(~+~, 0, zs)

Disco> sum [2i - 3 | i in [0 .. 10]]
```

(a)

$$\begin{aligned}\sum_{0\le i\le 10}(2i-3) &= 2\left(\sum_{0\le i\le 10} i\right) - \sum_{0\le i\le 10} 3\\ &= 2\frac{10(10+1)}{2} - 11\cdot 3\\ &= 110-33=77\end{aligned}$$

(b)

$$\begin{aligned}\sum_{1\le k\le 20}(5-k) &= \left(\sum_{1\le k\le 20} 5\right) - \left(\sum_{1\le k\le 20} k\right)\\ &= 20\cdot 5 - \frac{20\cdot 21}{2}\\ &= 100-210=-110\end{aligned}$$

(c)

$$\begin{aligned}\sum_{0\le j\le 50}(3j-7) &= 3\left(\sum_{0\le j\le 50} j\right) - \left(\sum_{0\le j\le 50} 7\right)\\ &= 3\frac{50\cdot 51}{2} - 51\cdot 7\\ &= 3468\end{aligned}$$

(d)

$$\begin{aligned}&7+10+13+16+19+\cdots+100\\ &= \sum_{0\le k\le 31} 3k+7\\ &= 3\left(\sum_{0\le k\le 31} k\right) + \left(\sum_{0\le k\le 31} 7\right)\\ &= 3\frac{31\cdot 32}{2} + 32\cdot 7\\ &= 1712\end{aligned}$$

(e)

$$\begin{aligned}\sum_{7\le i\le 29} i &= \left(\sum_{1\le i\le 29} i\right) - \left(\sum_{1\le i\le 6} i\right)\\ &= \frac{29\cdot 30}{2} - \frac{6\cdot 7}{2}\\ &= 414\end{aligned}$$

(f)

$$\begin{aligned}\sum_{0\le k\le 10}(2^k-1) &= \left(\sum_{0\le k\le 10} 2^k\right) - \left(\sum_{0\le k\le 10} 1\right)\\ &= \frac{2^{11}-1}{2-1} - 11\\ &= 2047 - 11 = 2036\end{aligned}$$

(g)

$$\begin{aligned}&1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \cdots + \frac{1}{3^7}\\ &= \sum_{0\le k\le 7}\left(\frac{1}{3}\right)^k\\ &= \frac{1-(1/3)^8}{1-1/3}\\ &= 3280/2187\end{aligned}$$