

## Discrete Math HW 9: Learning goals P4, P5

---

*P4: I can write the outline of a proof by (weak) induction or strong induction.*

**Exercise 1** Write the outline of a proof by (weak) induction of

$$\forall n \in \mathbb{N}. P(n).$$

**Exercise 2** Write the outline of a proof by strong induction of

$$\forall m \in \mathbb{N}. Q(m).$$

**Exercise 3** Prove by induction: for all natural numbers  $n$ ,

$$\sum_{1 \leq j \leq n+1} j \cdot 2^j = n \cdot 2^{n+2} + 2.$$

*P5: I can reproduce proofs by induction.*

On quizzes, you will be asked to reproduce one of the following proofs which we covered in class. For the purposes of this homework assignment, for each proof I suggest that you first study it, then put it aside for at least ten minutes, and then attempt to write out the proof without looking.

**Exercise 4** Prove by induction that for all natural numbers  $n$ ,

$$1 + 2 + \cdots + n = \frac{n(n+1)}{2}.$$

**Exercise 5** Prove by induction on  $n$  that  $2^n < n!$  for all  $n \geq 4$ .

**Exercise 6** Prove by induction that  $F_n \leq 2^{n-1}$  for all natural numbers  $n$  (where  $F_n$  denotes the  $n$ th Fibonacci number, defined by  $F_0 = 0$ ,  $F_1 = 1$ , and  $F_n = F_{n-1} + F_{n-2}$  for  $n \geq 2$ ).