Discrete Math HW 9: Learning goals P4, P5

P4: I can write the outline of a proof by (weak) induction.

Exercise 1 Write the outline of a proof by (weak) induction of

$$\forall n\!:\!\mathbb{N}.\ P(n).$$

Proof. By induction on n. We must prove $P(0)$, and $\forall k : \mathbb{N}.\ P(k) \to P(k+1)$.

> *Proof of* $P(0)$

> Let k be an arbitrary natural number, and suppose $P(k)$. We must prove $P(k+1)$.
>
> > *Proof of* $P(k+1)$*, using* $P(k)$
>
> Therefore, since k was arbitrary and we proved $P(k+1)$ while assuming $P(k)$, therefore $\forall k\!:\!\mathbb{N}.\ P(k) \to P(k+1)$.

Therefore, by induction, $P(n)$ is true for all natural numbers n. □

Exercise 2 Prove by induction: for all natural numbers n,

$$\sum_{1 \le j \le n+1} j \cdot 2^j = n \cdot 2^{n+2} + 2.$$

Proof. Let $Q(n)$ be the proposition

$$Q(n) \equiv \left(\sum_{1 \le j \le n+1} j \cdot 2^j = n \cdot 2^{n+2} + 2 \right).$$

We will show $\forall n\!:\!\mathbb{N}.\ Q(n)$ by induction.

> The base case, $Q(0)$, says
>
> $$\sum_{1 \le j \le 1} j \cdot 2^j = 0 \cdot 2^{n+2} + 2.$$
>
> Both sides are equal to 2, so this is true.

Now let k be an arbitrary natural number, and suppose $Q(k)$ is true, that is,

$$\sum_{1 \le j \le k+1} j \cdot 2^j = k \cdot 2^{k+2} + 2 \qquad (\text{KNOW}).$$

We will show $Q(k+1)$, that is,

$$\sum_{1 \le j \le k+2} j \cdot 2^j = (k+1) \cdot 2^{k+3} + 2 \qquad (\text{WANT}).$$

$$\sum_{1 \le j \le k+2} j \cdot 2^j$$

$=$ { pull the $j = k+2$ term out of the sum }

$$(k+2) \cdot 2^{k+2} + \sum_{1 \le j \le k+1} j \cdot 2^j$$

$=$ { assumption }

$$(k+2) \cdot 2^{k+2} + k \cdot 2^{k+2} + 2$$

$=$ { factor out 2^{k+2} }

$$((k+2)+k) \cdot 2^{k+2} + 2$$

$=$ { algebra }

$$(2k+2) \cdot 2^{k+2} + 2$$

$=$ { algebra }

$$(k+1) \cdot 2^{k+3} + 2$$

□

P5: I can reproduce proofs by induction.

On quizzes, you will be asked to reproduce one of the following proofs which we covered in class. For the purposes of this homework assignment, for each proof I suggest that you first study it, then put it aside for at least ten minutes, and then attempt to write out the proof without looking.

See class lecture notes for solutions.