Discrete Math HW 11: Learning goals N1, N3, N5 (solutions)

due Monday, April 20

N1: I can determine whether one integer is divisible by another, or whether a natural number up to 100 is prime.

Exercise 1 Determine which of the following numbers are prime, and give the prime factorization for each composite. Remember that to test n for primality, you only need to test for divisibility by primes $\leq \sqrt{n}$.

- 2 is prime.
- 3 is prime.
- 0 is not prime.
- 1 is not prime by definition.
- $10 = 2 \cdot 5$, so it is not prime.
- $51 = 3 \cdot 17$
- 53 is prime
- $86 = 2 \cdot 43$
- $87 = 3 \cdot 29$
- 89 is prime
- $91 = 7 \cdot 13$
- 101 is prime
- 103 is prime
- $1001 = 7 \cdot 11 \cdot 13$
- $1003 = 17 \cdot 59$
- $1005 = 3 \cdot 5 \cdot 67$
- $1007 = 19 \cdot 53$
- 1009 is prime

N3: I can compute the greatest common divisor of two natural numbers using the Euclidean Algorithm.

Exercise 2 Use the Euclidean Algorithm to compute each of the following. Be sure to show the steps of the process, not just the final result.

(a) $\gcd(1,5) = \gcd(5,1) = \gcd(1,0) = 1$

(b) $\gcd(123,277) = \gcd(277,123) = \gcd(123,31) = \gcd(31,30) = \gcd(30,1) = \gcd(1,0) = 1$

(c) $\gcd(78,104) = \gcd(104,78) = \gcd(78,26) = \gcd(26,0) = 26$

(d) $\gcd(88,72) = \gcd(72,16) = \gcd(16,8) = \gcd(8,0) = 8$

Exercise 3 Write a Disco function to find the GCD of two natural numbers using the Euclidean algorithm.

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gcd : (N * N) -> N
gcd(a,0) = a
gcd(a,b) = gcd(b, a mod b)
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$\gcd(518303142726377580, 169429189188136020) = 8580$

N5: I can solve modular equivalences in one variable involving addition, subtraction, and multiplication by a constant.

Exercise 4 Solve each of the following equivalences for x. Express your answers in the form $x \equiv_m r$ where $0 \le r < m$.

1. $x + 12 \equiv_7 99$
   $\leftrightarrow$ { subtract 12 from both sides }
   $x \equiv_7 87$
   $\leftrightarrow$ { reduce 87 modulo 7 }
   $x \equiv_7 3$

2. $27x + 27 \equiv_{13} 2727$
   $\leftrightarrow$ { reduce 27 and 2727 mod 13 }
   $x + 1 \equiv_{13} 10$
   $\leftrightarrow$ { subtract 1 from both sides }
   $x \equiv_{13} 9$

3. $2x - 12 \equiv_8 x + 7$

$\leftrightarrow$ { add 12 to both sides }

$2x \equiv_8 x + 19$

$\leftrightarrow$ { subtract x from both sides }

$x \equiv_8 19$

$\leftrightarrow$ { reduce 19 modulo 8 }

$x \equiv_8 3$

4. $77x + 15 \equiv_7 5 - 22x$

$\leftrightarrow$ { reduce 77 and 22 modulo 7 }

$0x + 15 \equiv_7 5 - x$

$\leftrightarrow$ { subtract 5 from both sides }

$10 \equiv_7 -x$

$\leftrightarrow$ { multiply both sides by -1 }

$-10 \equiv_7 x$

$\leftrightarrow$ { symmetry, and reduce -10 modulo 7 }

$x \equiv_7 4$