# *Discrete Math HW 10: Learning goals N1, N2 (solutions)*

*due Monday, April 13*

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*N1: I can determine whether one integer is divisible by another, or whether a natural number up to 100 is prime.*

**Exercise 1** Determine which of the following divisibility relationships hold.

- $2 \mid 90$: yes, since $2 \cdot 45 = 90$
- $3 \mid 90$: yes, since $3 \cdot 30 = 90$
- $4 \mid 90$: no, there is no integer $k$ such that $4k = 90$
- $5 \mid 10$: yes, $5 \cdot 2 = 10$
- $10 \mid 5$: no, $10k \neq 5$ for any integer $k$
- $10 \mid -10$: yes, $10 \cdot (-1) = 10$
- $0 \mid 6$: no, $0k \neq 6$ for any integer $k$
- $6 \mid 0$: yes, $6 \cdot 0 = 0$
- $0 \mid 0$: yes, $0 \cdot 27 = 0$
- $247 \mid 13585$: yes, $247 \cdot 55 = 13585$
- $(-2) \mid 4$: yes, $(-2) \cdot 2 = 4$
- $2 \mid (-4)$: yes, $2 \cdot (-2) = 4$
- $(-4) \mid 2$: no, $-4k \neq 2$ for any integer $k$

**Exercise 2** List all the positive integer divisors of 60.

The positive integer divisors of 60 are

$$1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60.$$

**Exercise 3** Prove that the divisibility relation is transitive: that is, for all $a, b, c \in \mathbb{Z}$, if $a \mid b$ and $b \mid c$, then $a \mid c$.

*Proof.* Let $a, b, c \in \mathbb{Z}$ be arbitrary, and suppose $a \mid b$ and $b \mid c$. That means there exist integers $j$ and $k$ such that $ja = b$ and $kb = c$. Then

$c$
$=$ { substitute $c = kb$ }
$kb$
$=$ { substitute $b = ja$ }
$k(ja)$
$=$ { multiplication is associative }
$(kj)a$

$kj$ is an integer since both $j$ and $k$ are. Thus, since $c$ is equal to an integer times $a$, by definition $a \mid c$. □

*N2: I can calculate quotients and remainders according to the Division Algorithm.*

**Exercise 4** Calculate each of the following quotients and remainders.

- $60 \textbf{ div } 12 = 5$
- $60 \textbf{ mod } 12 = 0$
- $60 \textbf{ div } 7 = 8$
- $60 \textbf{ mod } 7 = 4$
- $0 \textbf{ div } 12 = 0$
- $0 \textbf{ mod } 12 = 0$
- $12983 \textbf{ div } 527 = 24$
- $12983 \textbf{ mod } 527 = 335$
- $(-25) \textbf{ div } 7 = -4$
- $(-25) \textbf{ mod } 7 = 3$

*N4: I can solve modular equivalences in one variable involving addition, subtraction, and multiplication by a constant.*

**Exercise 5** Solve each of the following equivalences for $x$. Express your answers in the form $x \equiv_m r$ where $0 \leq r < m$.

1. $x + 12 \equiv_7 99$
   $\leftrightarrow$ { subtract 12 from both sides }
   $x \equiv_7 87$
   $\leftrightarrow$ { reduce 87 modulo 7 }

$x \equiv_7 3$

2. $27x + 27 \equiv_{13} 2727$
$\leftrightarrow$ { reduce 27 and 2727 mod 13 }
$x + 1 \equiv_{13} 10$
$\leftrightarrow$ { subtract 1 from both sides }
$x \equiv_{13} 9$

3. $2x - 12 \equiv_8 x + 7$
$\leftrightarrow$ { add 12 to both sides }
$2x \equiv_8 x + 19$
$\leftrightarrow$ { subtract $x$ from both sides }
$x \equiv_8 19$
$\leftrightarrow$ { reduce 19 modulo 8 }
$x \equiv_8 3$

4. $77x + 15 \equiv_7 5 - 22x$
$\leftrightarrow$ { reduce 77 and 22 modulo 7 }
$0x + 15 \equiv_7 5 - x$
$\leftrightarrow$ { subtract 5 from both sides }
$10 \equiv_7 -x$
$\leftrightarrow$ { multiply both sides by $-1$ }
$-10 \equiv_7 x$
$\leftrightarrow$ { symmetry, and reduce $-10$ modulo 7 }
$x \equiv_7 4$