Discrete Math HW 7: Learning goals F3, P3

due Monday, March 16

F3: I can determine whether a given function is 1-1 and/or onto.

Exercise 1 For each of the following functions, determine whether the function is injective (1-1), surjective (onto), both, or neither, and justify your assertions. Feel free to use Disco to help explore the behavior of these functions, though you are not required to do so. For full credit, complete at least 4.

(a) $f : \mathbb{Z} \to \mathbb{Z}$; $f(x) = x - 1$

(b) $f : \mathbb{N} \to \mathbb{Z}$; $f(x) = x - 1$

(c) $f : \mathbb{Z} \to \mathbb{Z}$; $f(x) = x^3$

(d) $f : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$; $f(a, b) = a + b$

(e) $f : \mathbb{Q} \to \mathbb{Q}$; $f(x) = 5x + 7$

(f) $f : \mathbb{N} \to \mathbb{Z}$; $f(x) = x$

(g) $f : \mathbb{Z} \to \mathbb{N}$; $f(x) = |x|$

P3: I can reproduce proofs about 1-1, onto, and bijective functions.

Exercise 2 Prove: if $f : A \to B$ and $g : B \to C$ are both one-to-one, then the composite function $g \circ f : A \to C$, defined by $(g \circ f)(a) = g(f(a))$, is also one-to-one.

Exercise 3 Prove: if $f : A \to B$ and $g : B \to C$ are both onto, then the composite function $g \circ f : A \to C$, defined by $(g \circ f)(a) = g(f(a))$, is also onto.