Discrete Math HW 6: Learning goals F1, F2

This homework assignment has no explicit Disco exercises, but you are of course welcome to use Disco to help you play around with examples if you find it helpful. For example, on Exercise 1 part 7, although you cannot create infinite sets in Disco, you could use Disco to help you construct *part of* the given set, which might help you look for patterns:

```
Disco> {((a,b),(c,d)) | a in {1..5}, b in {1..5}, c in {1..5}, d in {1..5}, a + d == b + c}
```

F1: I can determine whether a relation is reflexive, transitive, symmetric, antisymmetric, or an equivalence, and give examples of relations with these properties.

Exercise 1 For each relation below, state whether it is reflexive, transitive, symmetric, and/or antisymmetric. Give brief reasoning/justification/proof for each. For credit, do **at least 4**.

1. The $<$ relation on $\mathbb{Z}$, that is, $\{(a,b) \mid a \in \mathbb{Z}, b \in \mathbb{Z}, a < b\}$
2. The $\leq$ relation on $\mathbb{Z}$, that is, $\{(a,b) \mid a \in \mathbb{Z}, b \in \mathbb{Z}, a \leq b\}$
3. $\{(x,y) \mid (\text{Odd}(x) \wedge \text{Odd}(y)) \vee (\text{Even}(x) \wedge \text{Even}(y))\}$
4. $\{(x,y) \mid (\text{Odd}(x) \wedge \text{Even}(y)) \vee (\text{Even}(x) \wedge \text{Odd}(y))\}$
5. $\{(a,b) \mid a \wedge b \equiv \mathit{True}\}$
6. $\{(x,x) \mid x \in \mathbb{N}, \frac{x}{x} = 1\}$
7. $\{((a,b),(c,d)) \mid a,b,c,d \in \mathbb{N}, a+d = b+c\}$
8. $\{((p,q),(r,s)) \mid p,q,r,s \in \mathbb{Z}, ps = qr\}$
9. The empty relation, $\varnothing$
10. The complete relation on $\mathbb{Z}$, $\{(a,b) \mid a,b \in \mathbb{Z}\}$

Exercise 2 Give an example of a relation with each given set of properties. Justify your examples.

1. Transitive but not reflexive
2. Reflexive and transitive but not symmetric
3. An equivalence relation

F2: I can describe the equivalence classes of a given equivalence relation.

Exercise 3 Each of the relations below is an equivalence relation (you can check this if you like, but you may assume it is true). For each relation, state how many equivalence classes there are, and describe them (either informally, via words + examples, or formally, *e.g.* via set builder notation).

1. $\{(x, x) \mid x \in \mathbb{N}\}$
2. $\{(a, b) \mid a \in \mathbb{Z}, b \in \mathbb{Z}, (a - b) \text{ is evenly divisible by } 3\}$
3. $\{(x, y) \mid x \in \mathbb{N}, y \in \mathbb{N}, (x = 1) \leftrightarrow (y = 1)\}$
4. $\{(w, v) \mid w, v \in \text{English words}, w \text{ and } v \text{ start with the same letter}\}$
5. $\{(s, t) \mid s, t \in \mathbb{N}, \lfloor s/2 \rfloor = \lfloor t/2 \rfloor\}$

$\lfloor x/2 \rfloor$ means to divide x by 2 and round the result down to the nearest integer. For example, $\lfloor 4/2 \rfloor = \lfloor 5/2 \rfloor = 2$.