Discrete Math Challenge HW 8 (2 points)

Suppose p and q are primes, and let $n = pq$.

(a) How many positive integers less than or equal to n are divisible by p? How many are divisible by q?

(b) Use the subtraction rule (*i.e.* the Principle of Inclusion-Exclusion) to find the number of positive integers less than or equal to n which share a common factor with n.

(c) How many positive integers less than or equal to n are relatively prime to (*i.e.* do *not* share any common factors with, *i.e.* have a gcd of 1 with) n?

(d) The *Euler totient function* $\varphi(n)$ denotes the number of positive integers $\leq n$ which are relatively prime to n. For example, $\varphi(12) = 4$ since 1, 5, 7 and 11 are the only positive integers ≤ 12 which share no common factors with 12. Use your answer to the previous part to show that $\varphi(pq) = \varphi(p) \cdot \varphi(q)$.