

Discrete Math Challenge HW 5 (2 points)

Suppose that $f : A \rightarrow B$ and $g : B \rightarrow A$ are functions. This question explores the idea of showing that f is a bijection by demonstrating that it has an inverse.

- (a) Prove that if $\forall a : A. g(f(a)) = a$, then f is injective (1-1).
- (b) Prove that if $\forall b : B. f(g(b)) = b$, then f is surjective (onto).
- (c) Finally, prove the converse, that is, if $f : A \rightarrow B$ is 1-1 and onto, then it has an inverse.

Hint: use the fact that f is onto to define an appropriate function g ; then show that $g(f(a)) = a$ and $f(g(b)) = b$.