Discrete Math Challenge HW 2 (1 point)

Let the predicates F and R be defined as follows:

$$F(p, b) = \text{“}p\text{'s favorite book is } b\text{”}$$
$$R(p, b) = \text{“}p \text{ has read book } b\text{”}$$

Now consider the sentence

> Everyone either has a favorite book, or has never read any books.

(a) Encode this sentence in propositional logic using the predicates F and R.

(b) Use a sequence of logical equivalences to explain why the above sentence is logically equivalent to the sentence

> Everyone who has read at least one book has a favorite book.

(c) Let the predicate Q be defined as

$$Q(p, b_1, b_2) = p \text{ likes } b_1 \text{ at least as much as } b_2.$$

In other words, $Q(p, b_1, b_2)$ holds when p likes b_1 better than b_2, or likes them equally. Now suppose we define $F(p, b)$ (“p's favorite book is b”) to mean “p likes b at least as much as any book they have read”. Show how to encode this definition for F in terms of R and Q.