

Discrete Math Challenge HW 3 (2 points)

Recall from class that a rational number is defined as one which can be expressed as the ratio of two integers. That is,

$$\text{Rational}(x) \equiv \exists p:\mathbb{Z}. \exists q:\mathbb{Z}. (x = p/q).$$

- (a) Prove that the sum of a rational number and an irrational number must be irrational.
- (b) Prove that the product of a nonzero rational number and an irrational number must be irrational.
- (c) Give an example showing that it is possible for the sum of two irrational numbers to be rational.
- (d) Prove that $\sqrt{2} + \sqrt{3}$ is in fact irrational.

Hint: simplify $(\sqrt{2} + \sqrt{3})^2$ and use the results from the previous exercises to show that $(\sqrt{2} + \sqrt{3})^2$ must be irrational; then explain why this shows $\sqrt{2} + \sqrt{3}$ must be irrational as well. In class, we proved that $\sqrt{2}$ is irrational. You may assume that $\sqrt{3}$ is also irrational. In fact, $\sqrt{n}$ is always irrational whenever n is not a perfect square, though we won't be able to prove this until later in the course.