Discrete Math Challenge HW 8 (2 points)

As usual, the Fibonacci numbers F_n are defined by the recurrence:

$$
\begin{aligned}
F_0 &= 0 \\
F_1 &= 1 \\
F_n &= F_{n-1} + F_{n-2} \quad (n \geq 2)
\end{aligned}
$$

In class, we proved that $F_n \leq 2^{n-1}$ for all n. Prove that $(3/2)^n < F_n$ past a certain point. That is,

$$\exists c : \mathbb{N}.\ \forall n : \mathbb{N}.\ (n \geq c) \rightarrow (3/2)^n < F_n.$$

Together, these two results show that the Fibonacci numbers grow exponentially, since $(3/2)^n < F_n < 2^n$ for big enough n.

(In fact, the Fibonacci numbers grow proportionally to φ^n, where $\varphi = (1 + \sqrt{5})/2 \approx 1.618\ldots$.)