

## Discrete Math Challenge HW 9 (2 points)

---

Write a Disco function implementing the *extended* Euclidean algorithm, which finds not only the GCD of  $a$  and  $b$ , but also integers  $s$  and  $t$  such that  $sa + tb = \gcd(a, b)$ . That is, define a function  $egcd : \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{Z} \times \mathbb{Z} \times \mathbb{N}$  such that if  $egcd(a, b) = (s, t, g)$ , then  $sa + tb = g$  and  $g$  is the GCD of  $a$  and  $b$ . Some hints:

- Start by writing a recursive helper function

$$egcdH : (\mathbb{Z} \times \mathbb{Z} \times \mathbb{N}) \times (\mathbb{Z} \times \mathbb{Z} \times \mathbb{N}) \rightarrow \mathbb{Z} \times \mathbb{Z} \times \mathbb{N}$$

which takes the previous and current rows of the table and returns the last row of the table (the row containing the GCD of  $a$  and  $b$ ).

- Then implement  $egcd : \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{Z} \times \mathbb{Z} \times \mathbb{N}$  simply by calling  $egcdH$  with the right values for the starting rows.