Discrete Math Challenge HW 7 (3 points)

This challenge concerns a variant of the *Ackermann function* (originally due to R. C. Buck), defined on pairs of natural number inputs as follows:

$$A(m,n) = \begin{cases} 2n & \text{if } m = 0 \\ 0 & \text{if } m \geq 1 \text{ and } n = 0 \\ 2 & \text{if } m \geq 1 \text{ and } n = 1 \\ A(m-1, A(m, n-1)) & \text{if } m \geq 1 \text{ and } n \geq 2 \end{cases}$$

(a) Find $A(1,0)$, $A(0,1)$, $A(1,1)$, and $A(2,2)$.

(b) Prove that $A(m,2) = 4$ for all natural numbers m.

(c) Prove that $A(1,n) = 2^n$ for all $n \geq 1$.

(d) Find each of the following values.

You might be able to use Disco to find some of them; for others, you can use the above theorems to help you compute a result on paper, since the result may be too big for Disco (or any computer) to handle.

(i) $A(2,3)$

(ii) $A(3,3)$

(iii) $A(3,4)$

(*Hint:* feel free to leave your answer in a convenient algebraic form rather than expanding it out into its actual decimal digits.)