

Discrete Math HW 6: Learning goals F3, R1, R3
due Friday, April 4

F3: I can determine whether a given function is 1-1 and/or onto.

Exercise 1 For each of the following functions, determine whether the function is injective (1-1), surjective (onto), both, or neither, and justify your assertions. Feel free to use Disco to help explore the behavior of these functions, though you are not required to do so. For full credit, complete at least 4.

(a) $f : \mathbb{Z} \rightarrow \mathbb{Z}; f(x) = x - 1$

(b) $f : \mathbb{N} \rightarrow \mathbb{Z}; f(x) = x - 1$

(c) $f : \mathbb{Z} \rightarrow \mathbb{Z}; f(x) = x^3$

(d) $f : \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{N}; f(a, b) = a + b$

(e) $f : \mathbb{Q} \rightarrow \mathbb{Q}; f(x) = 5x + 7$

(f) $f : \mathbb{N} \rightarrow \mathbb{Z}; f(x) = x$

(g) $f : \mathbb{Z} \rightarrow \mathbb{N}; f(x) = |x|$

R1: I can compute the terms of a sequence defined via a recurrence.

Exercise 2 List the first five terms of each sequence defined below.

(a)

$$a_0 = 1$$
$$a_n = a_{n-1} + 3 \quad (n \geq 1)$$

(b)

$$a_2 = 7$$
$$a_n = a_{n-1} - 1 \quad (n \geq 3)$$

(c)

$$p_0 = 0$$
$$p_n = 3p_{n-1} - 1 \quad (n \geq 1)$$

(d)

$$p_0 = 0$$

$$p_n = p_{n-1} + n^2 \quad (n \geq 1)$$

(e)

$$p_0 = 1$$

$$p_1 = 1$$

$$p_n = 2p_{n-1} + p_{n-2} \quad (n \geq 2)$$

R3: I can evaluate sums involving arithmetic and geometric series.

Exercise 3 Evaluate each of the following sums. Show your work. For credit, complete at least 4.

(a)

$$\sum_{0 \leq i \leq 10} (2i - 3)$$

(b)

$$\sum_{1 \leq k \leq 20} (5 - k)$$

(c)

$$\sum_{0 \leq j \leq 50} (3j - 7)$$

(d)

$$7 + 10 + 13 + 16 + 19 + \cdots + 100$$

(e)

$$\sum_{7 \leq i \leq 29} i$$

(f)

$$\sum_{0 \leq k \leq 10} (2^k - 1)$$

(g)

$$1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \cdots + \frac{1}{3^7}$$

