Discrete Math HW 5: Learning goal F1

F1: I can determine whether a relation is reflexive, transitive, symmetric, antisymmetric, or an equivalence, and give examples of relations with these properties.

Exercise 1 For each relation below, state whether it is reflexive, transitive, symmetric, and/or antisymmetric. Give brief reasoning/justification/proof for each.

For credit, do at least 6.

1. The $<$ relation on $\mathbb{Z}$, that is, $\{(a,b) \mid a \in Z, b \in Z, a < b\}$
2. The $\leq$ relation on $\mathbb{Z}$, that is, $\{(a,b) \mid a \in Z, b \in Z, a \leq b\}$
3. $\{(x,y) \mid (\text{Odd}(x) \wedge \text{Odd}(y)) \vee (\text{Even}(x) \wedge \text{Even}(y))\}$
4. $\{(x,y) \mid (\text{Odd}(x) \wedge \text{Even}(y)) \vee (\text{Even}(x) \wedge \text{Odd}(y))\}$
5. $\{(a,b) \mid a \wedge b \equiv \mathit{True}\}$
6. $\{(x,x) \mid x \in \mathbb{N}, \frac{x}{x} = 1\}$
7. $\{((a,b),(c,d)) \mid a,b,c,d \in \mathbb{N}, a+d = b+c\}$
8. $\{((p,q),(r,s)) \mid p,q,r,s \in \mathbb{Z}, ps = qr\}$
9. The empty relation, $\varnothing$
10. The complete relation on $\mathbb{Z}$, $\{(a,b) \mid a,b \in \mathbb{Z}\}$

Exercise 2 Give an example of a relation with each given set of properties.

1. Transitive but not reflexive
2. Reflexive and transitive but not symmetric
3. An equivalence relation