Discrete Math HW 4: Learning goals S1–S3, P2

S1: I can state the definitions, and determine membership, of standard sets such as $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, *and* $\mathbb{R}$.

Exercise 1 Name three examples of numbers which are members of $\mathbb{Z}$ but not $\mathbb{N}$.

Exercise 2 Name three examples of numbers which are in $\mathbb{Q}$ but not $\mathbb{Z}$.

S2: I can evaluate and construct sets using union, intersection, difference, and complement of sets, and sets defined via set comprehension notation.

Exercise 3 Let A, B, and C represent the following sets:

$$A = \{5, 8, 13, 20\}$$
$$B = \{n \mid n \in \mathbb{N} \wedge n < 10\}$$
$$C = \{10k \mid k \in \mathbb{Z}\}$$

Write out the elements in the sets corresponding to each expression below.

(a) $A - B$

(b) $B \cap C$

(c) $A \cap B \cap C$

(d) $\{x \mid (x \in C \cup A) \wedge (|x| \leq 10)\}$

S3: I can list all the elements in a power set, Cartesian product, or disjoint union of sets, or count them without listing them all.

Exercise 4 List all the elements of $\mathcal{P}(\{3, 5, 7\})$.

Exercise 5 List all the elements of $\mathcal{P}(\{1, 2, 3, 4\})$.

Exercise 6 List all the elements of $\{1, 2\} \times \{-3, -4, -5\}$.

Exercise 7 List all the elements of $\{A\} \times \{B, C, D, E\}$.

Exercise 8 List all the elements of $\{A, B, C\} \times \{X, Y, Z\}$.

Exercise 9 Evaluate: $|\mathcal{P}(\{1, \ldots, 5\})|$.

Exercise 10 Evaluate: $|\{A, B, C, \ldots, Z\} \times \{1, 2, 3, \ldots, 10\}|$.

P2: I can write proofs about sets, set operations, and the subset relation.

Do at least one of the following exercises.

Exercise 11 Prove that for all sets S and T,

$$\overline{S \cap T} = \overline{S} \cup \overline{T}.$$

Exercise 12 Prove that for all sets S, T, and U,

$$S \cup (T \cap U) = (S \cup T) \cap (S \cup U).$$