Discrete Math HW problems: Learning goals L1, L5

L1: I can make truth tables for propositional logic expressions involving TRUE, FALSE, AND, OR, NOT, IMPLIES, IFF, and variables.

Exercise 1 Suppose that p and q are true propositions, and r is false. Evaluate whether each of the following propositions is true or false, and check your answers using Disco. You can first paste the following into the text editor box to define p, q, and r:

```
p : Bool
p = T
q : Bool
q = T
r : Bool
r = F
```

Now, for example, to evaluate $p \wedge r$ you can just type `p /\ r` at the Disco prompt (remember that you can also use `and` or `&&` instead of `/\`).

(a) $p \wedge (q \vee r) \wedge \neg r$

(b) $(r \to q) \vee (q \to r)$

(c) $((r \to r) \to r) \to r$

(d) $(p \vee \neg p) \wedge (q \vee \neg q) \wedge (r \vee \neg r)$

Exercise 2 For each of the following, *either*:

- use a truth table to show that the two expressions are logically equivalent for all possible truth values of the propositional variables, *or*
- give an example of specific truth values for which the two expressions are different.

1. $(Q \to \neg P) \stackrel{?}{\equiv} (P \leftrightarrow Q)$
2. $(P \vee Q) \to R \stackrel{?}{\equiv} (P \to R) \wedge (Q \to R)$
3. $(P \to Q) \to R \stackrel{?}{\equiv} P \to (Q \to R)$

Exercise 3 Construct a truth table for the proposition $(P \wedge Q) \leftrightarrow (R \vee \neg Q)$.

L5: I can formalize English statements as propositional logic formulas, making appropriate use of logical connectives and nested quantifiers.

Exercise 4 Let the propositional variables p, q, and r be defined as follows:

$$p = \text{Unicorns are real.}$$
$$q = \text{Dragons are real.}$$
$$r = \text{Dr. Yorgey likes math.}$$

Using these variables, translate each of the following English sentences into formal propositional logic notation. For example, the sentence "Unicorns and dragons are real" could be translated as $p \wedge q$.

(a) Dr. Yorgey likes math, but unicorns are not real.

(b) Unicorns and dragons are real, and Dr. Yorgey likes math.

(c) Unicorns are real if Dr. Yorgey doesn't like math.

(d) Either dragons are real and Dr. Yorgey likes math, or unicorns are real and Dr. Yorgey doesn't like math.

(e) If either unicorns or dragons are real, then Dr. Yorgey likes math.

(f) Dragons either are or aren't real.