Discrete Math HW problems: Learning goals L1, L4 (solutions)

L1: I can make truth tables for propositional logic expressions involving TRUE, FALSE, AND, OR, NOT, IMPLIES, IFF, and variables.

Exercise 1 Suppose that p and q are true propositions, and r is false. Evaluate whether each of the following propositions is true or false, and check your answers using Disco.

All four are true, as shown below:

(a) $p \land (q \lor r) \land \neg r \equiv T \land (T \lor F) \land \neg F \equiv T \land T \land T \equiv T$

(b) $(r \to q) \lor (q \to r) \equiv (F \to T) \lor (T \to F) \equiv T \lor F \equiv T$

(c) $((r \to r) \to r) \to r \equiv ((F \to F) \to F) \to F \equiv (T \to F) \to F \equiv F \to F \equiv T$

(d) $(p \lor \neg p) \land (q \lor \neg q) \land (r \lor \neg r) \equiv (T \lor \neg T) \land (T \lor \neg T) \land (F \lor \neg F) \equiv T \land T \land T \equiv T$

Exercise 2 For each of the following, *either*:

- use a truth table to show that the two expressions are logically equivalent for all possible truth values of the propositional variables, *or*
- give an example of specific truth values for which the two expressions are different.

1. $(Q \to \neg P) \stackrel{?}{\equiv} (P \leftrightarrow Q)$

   These are not equivalent. For example, when Q is false and P is true, the left-hand side is true (since $F \to P$ is true for any P) but the right-hand side is false (since P and Q are not the same).

2. $(P \lor Q) \to R \stackrel{?}{\equiv} (P \to R) \land (Q \to R)$

   This is true, as shown by the fact that the last two columns in the below truth table are identical:

P	Q	R	$P \vee Q$	$P \to R$	$Q \to R$	$(P \vee Q) \to R$	$(P \to R) \wedge (Q \to R)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	T	T	T	T	T	T
T	F	F	T	F	T	F	F
F	T	T	T	T	T	T	T
F	T	F	T	T	F	F	F
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

3. $(P \to Q) \to R \stackrel{?}{\equiv} P \to (Q \to R)$

   This is false. For a counterexample, consider the case when P and R are both false (it does not matter what Q is). In that case, on the left we have

$$(P \to Q) \to R \equiv (F \to Q) \to F \equiv T \to F \equiv F,$$

   but on the right we have

$$P \to (Q \to R) \equiv F \to (Q \to F) \equiv T.$$

Exercise 3 Construct a truth table for the proposition $(P \wedge Q) \leftrightarrow (R \vee \neg Q)$.

P	Q	R	$P \wedge Q$	$R \vee \neg Q$	$(P \wedge Q) \leftrightarrow (R \vee \neg Q)$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	T	F
T	F	F	F	T	F
F	T	T	F	T	F
F	T	F	F	F	T
F	F	T	F	T	F
F	F	F	F	T	F

L4: I can formalize English statements as propositional logic formulas, making appropriate use of logical connectives and nested quantifiers.

Exercise 4 Let the propositional variables p, q, and r be defined as follows:

$$p = \text{Unicorns are real.}$$
$$q = \text{Dragons are real.}$$
$$r = \text{Dr. Yorgey likes math.}$$

Using these variables, translate each of the following English sentences into formal propositional logic notation. For example, the sentence "Unicorns and dragons are real" could be translated as $p \wedge q$.

(a) Dr. Yorgey likes math, but unicorns are not real.

$r \wedge \neg p$

Logically, the word "but" means the same as "and".

(b) Unicorns and dragons are real, and Dr. Yorgey likes math.

$p \wedge q \wedge r$

We could also wrote $(p \wedge q) \wedge r$ to reflect the way the phrases are grouped in the English sentence. However, since $\wedge$ is associative, the parentheses are not needed.

(c) Unicorns are real if Dr. Yorgey doesn't like math.

$\neg r \to p$

Be careful to get the direction of the implication correct! $p \to \neg r$ is not correct. When we say "X if Y", it means the same as "if Y, then X".

(d) Either dragons are real and Dr. Yorgey likes math, or unicorns are real and Dr. Yorgey doesn't like math.

$(q \wedge r) \vee (p \wedge \neg r)$

The parentheses are technically not required in this case—typically we say that AND has higher precedence than OR—but in my opinion, omitting them makes it difficult to read.

(e) If either unicorns or dragons are real, then Dr. Yorgey likes math.

$(p \vee q) \to r$

(f) Dragons either are or aren't real.

$q \vee \neg q$