The first page of your homework submission must be a cover sheet answering the following questions. Do not leave it until the last minute; it's fine to fill out the cover sheet before you have completely finished the assignment. Assignments submitted without a cover sheet, or with a cover sheet obviously dashed off without much thought at the last minute, will not be graded.

- How many hours would you estimate that you spent on this assignment?

- Explain (in one or two sentences) one thing you learned through doing this assignment.

- What is one thing you think you need to review or study more? What do you plan to do about it?

Question 1 (5 points)**.** Prove: for all $n \geq 1$, if G is a connected graph with n vertices and $n-1$ edges, then G has no cycles. (This is the third part of the proof from class, characterizing trees as having any two out of three properties.)

Question 2 (5 points)**.** Let $G = (V, E)$ be an undirected graph with n vertices, with no self-loops (that is, no edges of the form (v, v) from a vertex to itself). Show that if every vertex has degree at least $n/2$, the graph is connected. If it makes your proof easier, you may assume that n is even.

Question 3 (5 points)**.** Consider the family of undirected graphs $\mathcal{H}_k$ defined as follows. $\mathcal{H}_k$ has 2^k vertices labelled with the integers 0 through $2^k - 1$. Vertices u and v are connected by an edge if and only if the binary representations of u and v differ in exactly one bit position. For example, in $\mathcal{H}_4$, the vertices 5 and 13 are connected by an edge since $5 = 0101_2$ and $13 = 1101_2$ differ in the first bit position, but the rest of the bits are the same.

Consider doing a BFS in $\mathcal{H}_{10}$ starting at node 0. How many vertices are in L_6, that is, the sixth layer generated by the BFS? Give your answer together with either a proof, or the program you used to calculate the answer. Either approach will receive full credit. (*Hint* if you choose to write a program: to flip the `j`th bit of an integer `n`, you can use `n ^ (1 << j)`, that is, the bitwise XOR of `n` with the result of shifting 1 left `j` times, that is, $2^{\mathtt{j}}$. These operators are valid in many languages such as Java, Python, and C/C++.)

Question 4 (10 points)**.** On the website you will find a file called `graph.txt` which describes a large undirected graph. The first line of the file contains a single integer which is the number of edges in the graph. Each subsequent line of the file describes one (undirected) edge, and contains two space-separated strings which are the names of the two vertices at the endpoints of the edge.

Write a program (in a programming language of your choice) to find a shortest path from the vertex labelled with your first name to the vertex labelled `END` (if there are multiple shortest paths you can find any one of them). You should submit a text file containing the list of vertices along this shortest path, starting with your name and ending with `END`. Each vertex should be on a separate line. For example, my solution looks like this:

```
Brent
7runq3
ajbkfy
ra17g1
jk9ojb
k289nn
gjinu8
END
```

You should also turn in the code you used to find your path.