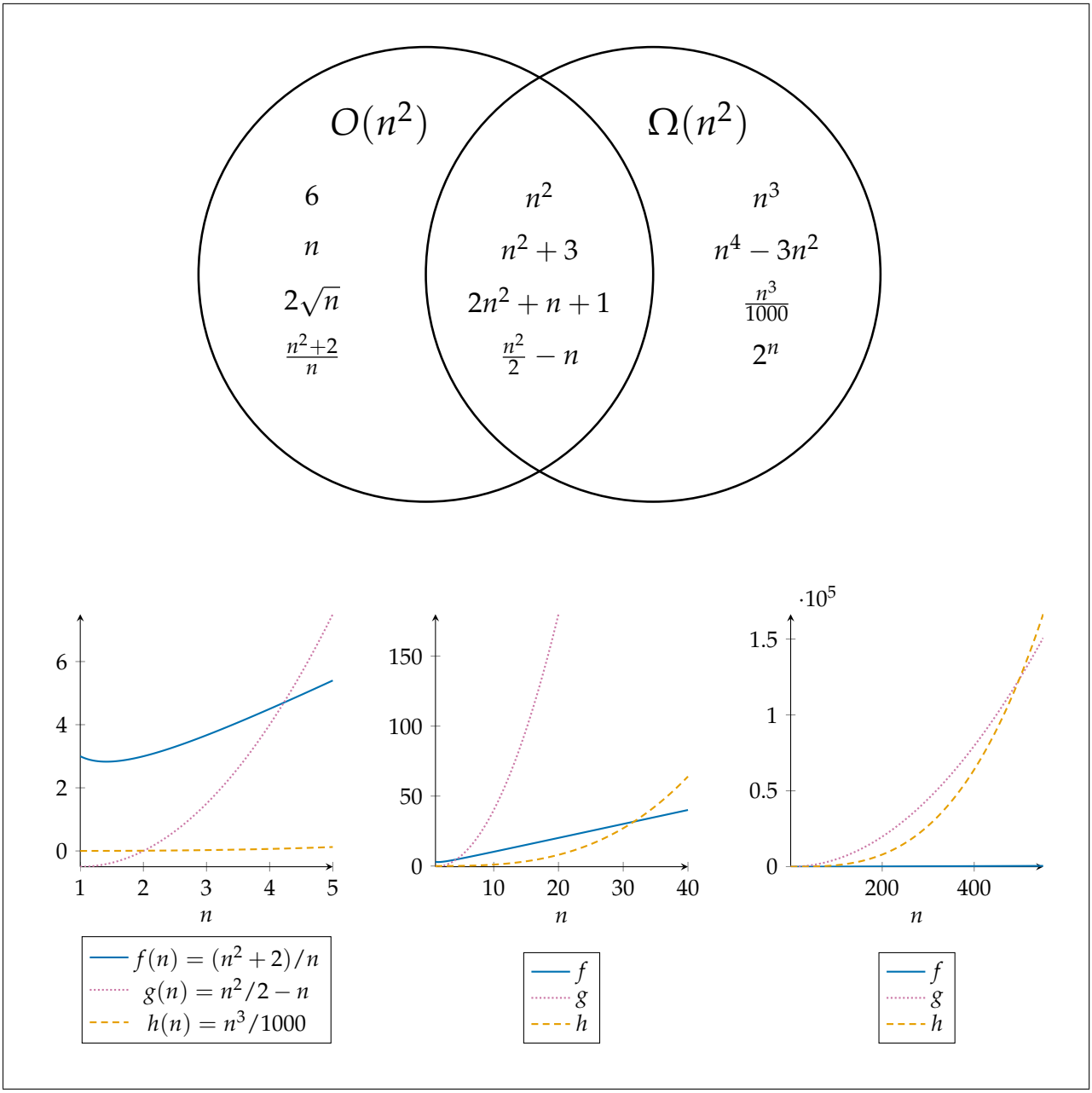

Model 1: Big-O and Big-Ω



Critical Thinking Questions I

Learning objective: Students will describe asymptotic behavior of functions using big-O, big-Θ, and big-Ω notation.

Important note: although your previous experience with big-O notation will certainly be helpful, I am not assuming that you remember anything in particular. When answering the following questions, as much as possible, try to rely on the information provided in Model 1 rather than on your memory.

1 Based on the Venn diagram in the model, say whether each function is $O(n^2)$, $\Omega(n^2)$, or both.

(a) $2\sqrt{n}$

(b) n^3

(c) $2n^2 + n + 1$

(d) 2^n

Consider the functions

$$f(n) = (n^2 + 2)/n,$$
$$g(n) = n^2/2 - n, \text{and}$$
$$h(n) = n^3/1000$$

for which graphs are shown in the model.

2 On each of the following intervals, list the functions f, g, and h from largest to smallest.

(a) $n \in [2, 4]$

(b) $n \in [5, 30]$

(c) $n \in [35, 450]$

3 Which function is largest, and which the smallest, at $n = 600$?

4 Does this relative order continue for all $n \geq 600$, or do the functions ever change places again? Justify your answer.

5 How do you think your answers to the previous questions relate to whether each of f, g, and h is $O(n^2)$, $\Omega(n^2)$, or both?

Say whether you think each of the following statements is true or false. Give a short justification for each answer.

6 If $f(n)$ is $O(n^2)$, then it has n^2 in its definition.

7 If $f(n)$ has n^2 in its definition, then $f(n)$ is $O(n^2)$.

8 If $f(n)$ is both $O(n^2)$ and $\Omega(n^2)$, then it has n^2 in its definition.

9 If $f(n) \leq n^2$ for all $n \geq 0$, then $f(n)$ is $O(n^2)$.

10 If $f(n)$ is $O(n^2)$, then $f(n) \leq n^2$ for all $n \geq 0$.

11 If $f(n) \leq n^2$ for all n that are sufficiently large, then $f(n)$ is $O(n^2)$.

12 If $f(n)$ is $O(n^2)$ and $g(n)$ is $\Omega(n^2)$, then $f(n) \leq g(n)$ for all $n \geq 0$.

13 Every function $f(n)$ is either $O(n^2)$ or $\Omega(n^2)$ (or both).

14 Using one or more complete English sentences and appropriate mathematical formalism, propose a definition of $O(n^2)$ by completing the following statement.

A function $f(n)$ is $O(n^2)$ if and only if...

Critical Thinking Questions II

15 In what way(s) do you think the definition of $\Omega(n^2)$ is similar to that of $O(n^2)$?

16 In what way(s) do you think it is different?

17 If a function is both $O(n^2)$ and $\Omega(n^2)$, we say it is $\Theta(n^2)$. For each of the below functions, say whether you think it is $\Theta(n^2)$. Justify your answers.

(a) $3n^2 + 2n - 10$

(b) $\dfrac{n^3 - 5}{n}$

(c) $\dfrac{n^3 - 5}{\sqrt{n}}$

(d) $(n+1)(n-2)$

(e) $n + n\sqrt{n}$

18 Do you think $n^2 \cdot \log_2 n$ is $O(n^2)$, $\Omega(n^2)$, or both? Why?