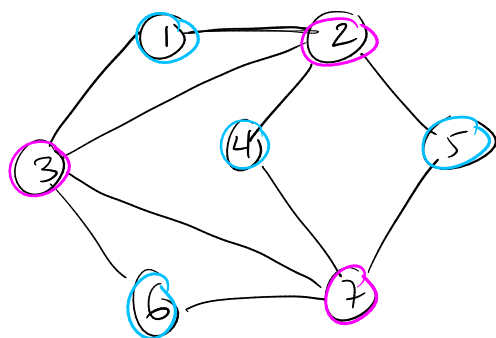


Hard (?) problems

Def: An independent set in an undirected graph $G = (V, E)$ is a subset of vertices $S \subseteq V$ such that no two vertices in S are adjacent.

e.g.



Independent sets:

$\{1, 6, 4, 5\}$

$\{3, 4, 5\}$

$\{2, 6\}$

$\{4\}$

$\{1, 4, 6\}$

$\emptyset$

Interesting Q's about independent sets?

— Largest ind. set? $\leftarrow$

— Total # of ind. sets? $\leftarrow$

Brute force?

— List every subset, check if each is independent. keep track of biggest.

$O(2^V \cdot V^2)$. — fundamentally, we don't know better algorithms.

Example 2: Satisfiability (SAT)

Def: A term is either a variable or the negation of a variable

e.g. $x_1, x_3, \neg x_5$
 $(\overline{x_5})$

Def: A clause is one or more terms combined with OR.

e.g. $x_1 \vee \overline{x_3} \vee x_4$

x_2

$x_1 \vee x_2 \vee x_3 \vee x_5 \vee \overline{x_7} \vee x_{10}$

Def'n A truth assignment assigns T or F to each variable.

$$g. \{x_1 \mapsto T, x_2 \mapsto F, x_3 \mapsto T\}.$$

A truth assignment satisfies a clause if it makes the clause true.

g. the above satisfies

$$\begin{aligned} & \underline{x_2} \vee \underline{x_1} \vee \underline{\overline{x_3}} \\ & F \vee T \vee F \equiv T. \end{aligned}$$

but not

$$x_2 \vee \overline{x_3} \vee \overline{x_1}$$

Def'n A truth assignment satisfies a collection of clauses if it satisfies all of them i.e. $C_1 \wedge C_2 \wedge \dots \wedge C_n$.

$$g. (x_1 \vee \overline{x_2}), (\overline{x_1} \vee \overline{x_3}), (x_2 \vee \overline{x_3})$$

$\{x_1 \mapsto T, x_2 \mapsto T, x_3 \mapsto F\}$ satisfies above clauses.

$$g. x_1, (x_3 \vee \overline{x_1}), (\overline{x_1} \vee \overline{x_3})$$

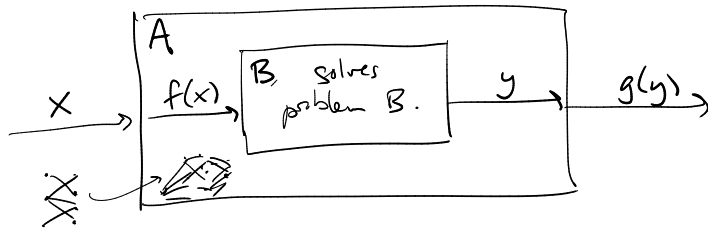
is not satisfiable, i.e. no truth assignment satisfies it.

Q (SAT) : Given a collection of clauses $C_1, \dots, C_k$ over the variables $\{x_1, \dots, x_n\}$, are the clauses satisfiable?

Brute force : try every truth assignment, $\Omega(2^n)$.

Variant : Same q, but where all clauses have exactly 3 terms (3-SAT).

How do we know if one problem is "harder than" another?



If A can be solved in this way using a black box that solves B , we say " A is reducible to B " and write $A \leq B$.

by Bipartite matching $\leq$ Max flow

SAT $\leq$ 3-SAT

3-SAT $\leq$ INDEPENDENT SET.