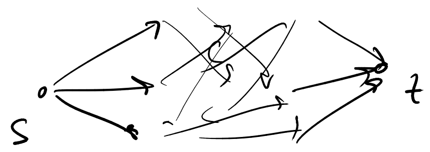
Flow Networks

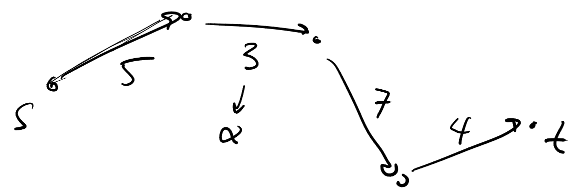
- Directed, weighted graph — weights = "capacity"
- Source node + Sink



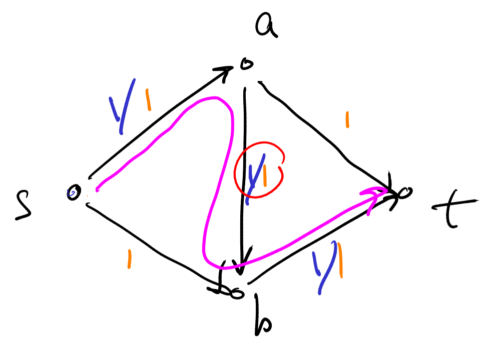
- Flow $f(e)$ each edge must be $\leq c(e)$ (capacity)
- At each vertex, incoming flow = outgoing flow.

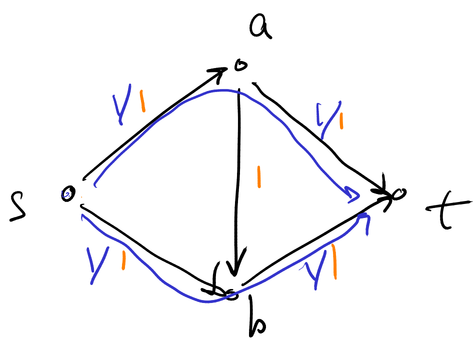
GreedyFlow(G, s, t):

while there exists an unsaturated path P
    from $s \to t$:
    $\alpha \leftarrow$ minimum remaining capacity along P
    for each edge $e \in P$:
        Add α to $f(e)$.



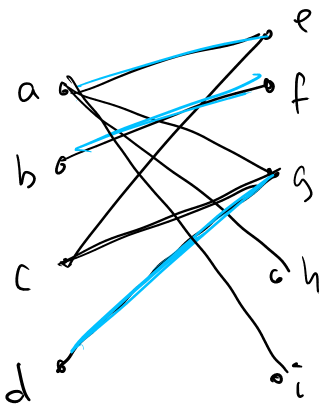
This does not work!



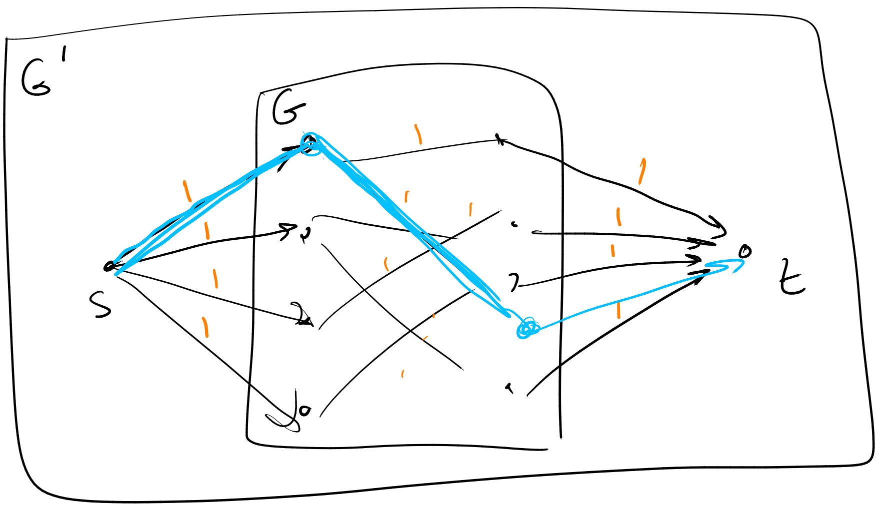


Example applications!

① Maximum bipartite matching.



Matching = set of edges ("matches") which do not share any endpoints.



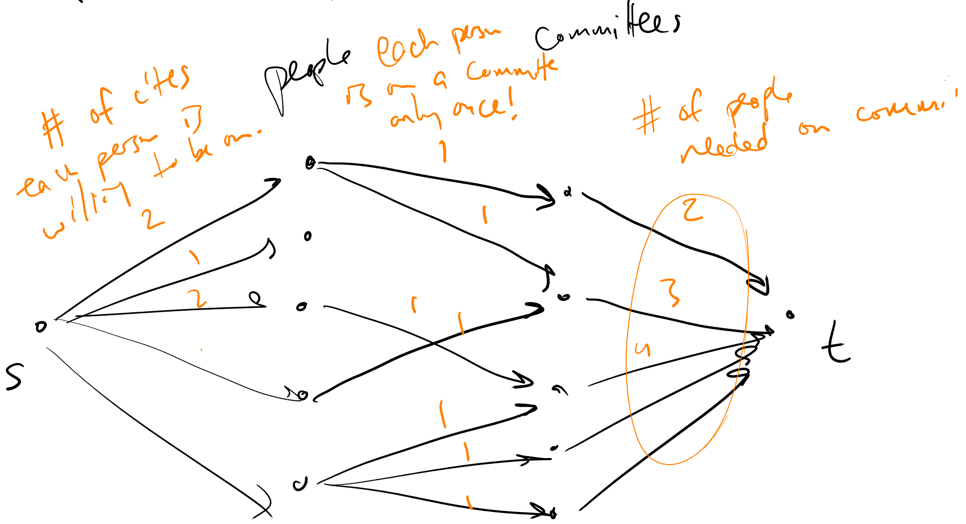
Claim: flows of value k on G' are in 1-1 correspondence with matchings of size k in G.

Why? Because of flow rules, any edge of G with flow on it cannot share endpoint w/ any other such edge. Hence, edges w/ 1 unit of flow are a valid matching + vice versa.

→ Max flow = max matching.

② Committee assignment

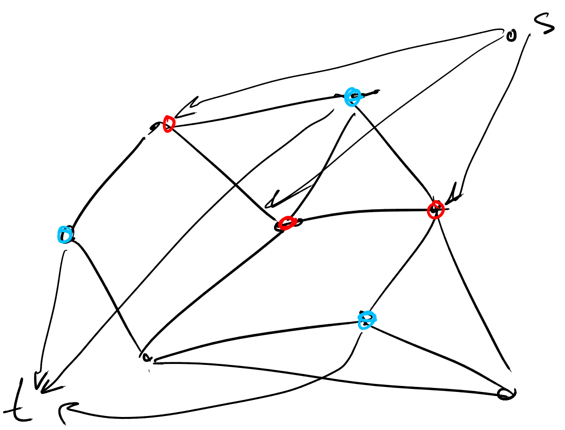
- People
- Committees
- Each person is only willing to be on certain committees, + has a max total # they are willing to be on.
- Each committee has a specific # of people needed.
- Q: is it possible to assign people to committees?



edge $p \to c$
= p is willing to be on committee c.

Others:

- Class / room / time final exam assignments.
- Evacuation planning.



- Baseball playoff elimination.
- others!