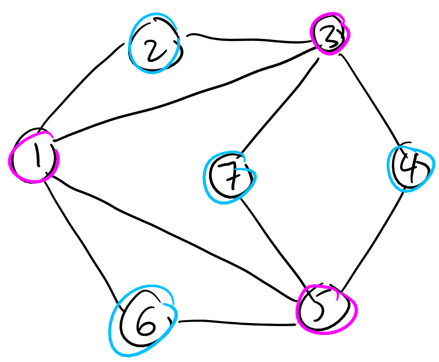
P vs NP!

Problems where we don't know better solution than brute force.

① Independent set.

Def'n Given an undirected graph $G=(V,E)$, an independent set is a subset of the vertices $S \subseteq V$ such that no two vertices in S are adjacent.

eg.



Independent sets

$\{2,4,6,7\}$

$\{1,7,4\}$

$\{1\}$

$\emptyset$

Interesting Q's:

- How many independent sets?
- Largest independent set? ←

Brute force to find largest ind. set?

- List all subsets — $O(2^V)$
- Check whether each is independent $O(V^2)$? $O(E)$?
- Remember biggest

$O(V^2 2^V)$ — don't know fundamentally better alg. than this.

"Decision problem" = problem w/ Y/N answer.

IND-SET : Given G, number k, is there an independent set in G of size $\geq k$?

② SAT (Satisfiability)

Def'n A term is either a variable or the negation of a variable

e.g. x_2 $\quad \neg x_3$ $\quad$ etc.

$(\overline{x_3})$

Def'n A clause is one or more terms combined with OR.

e.g. $x_1 \vee x_3 \vee \overline{x_5}$

x_2

$x_1 \vee x_2 \vee x_3 \vee x_4 \vee \overline{x_7} \vee x_8$

Def'n A truth assignment is an assignment of T or F to each variable.

e.g. $\{x_1 \mapsto T,\ x_2 \mapsto F,\ x_3 \mapsto T\}$.

A truth assignment satisfies a clause if it makes it true.

e.g. $\{x_1 \mapsto T,\ x_2 \mapsto F,\ x_3 \mapsto F,\ x_4 \mapsto F\}$

satisfies $x_1 \vee \overline{x_3} \vee x_4$

Def'n A truth assignment satisfies a collection of clauses $C_1, C_2, \dots, C_k$ if it satisfies all of them. i.e. $C_1 \wedge C_2 \wedge \dots \wedge C_k$.

e.g. $(x_1 \vee \overline{x_2}),\ (\overline{x_1} \vee \overline{x_3}),\ (x_2 \vee \overline{x_3})$

is satisfied by $\{x_1 \mapsto T,\ x_2 \mapsto T,\ x_3 \mapsto F\}$.

$x_1,\ (x_3 \vee \overline{x_1}),\ (\overline{x_3} \vee \overline{x_1})$

— not satisfiable.

Q: Given a collection of clauses, are they satisfiable? (SAT)

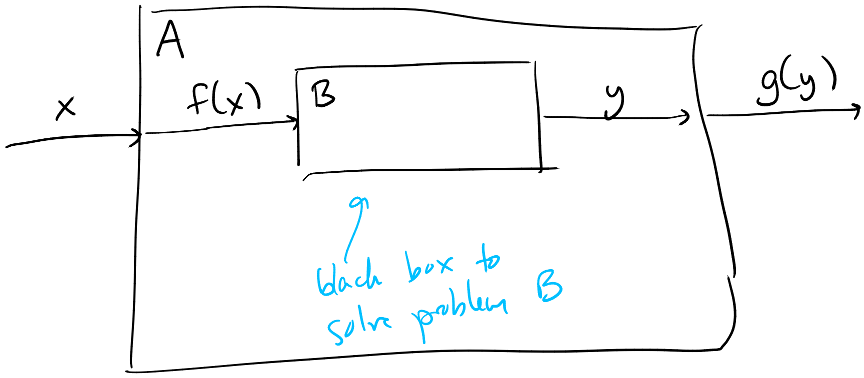
Brute force? Try all truth assignments — $\Omega(2^n)$
(n = # of variables)

<u>3-SAT</u>: Given a collection of clauses, each of which contains exactly 3 terms, are they satisfiable?

Turns out to be equivalently difficult as SAT.

How do we know which problems are harder than other problems?

Answer: reducibility.



If black box for B can be used to solve A (and f and g don't take too long), we say A is reducible to B and write

$$A \leq B$$

eg. SAT $\leq$ 3-SAT.
Bipartite matching $\leq$ Max flow

claim: 3-SAT $\leq$ IND-SET.