

Floyd-Warshall : all-pairs shortest paths.

Recall: Dijkstra solves the single-source shortest path problem,
& only for nonnegative edge weights.

Input: directed, weighted graph G .

Goal: $d[u, v] = \text{length of shortest path } u \rightsquigarrow v$.

- For now, assume no negative-weight cycles.
- But! F-W can detect presence of neg. wt. cycles.

Add a parameter!

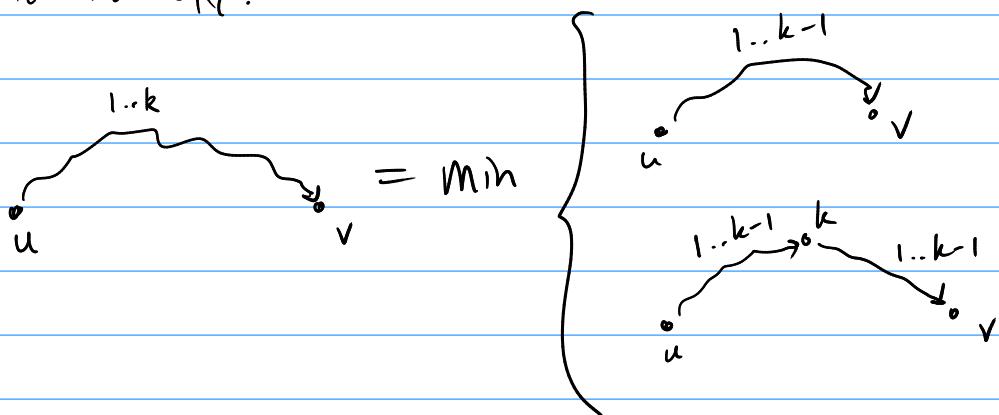
$d[u, v, k]$ = length of a shortest path $u \rightarrow v$
which only uses intermediate vertices from $\{1, \dots, k\}$.

Base cases?

$$d[u, v, 0] = \begin{cases} 0 & \text{if } u = v \\ w_{uv} & \text{if } (u, v) \in E \\ \infty & \text{if } (u, v) \notin E. \end{cases}$$

not allowed to use intermediate vertices

Induction step?



$$d[u, v, k] = \min(d[u, v, k-1], d[u, k, k-1] + d[k, v, k-1])$$

Initialize $d[u, v]$ according to base cases

for k in $1..n$:

 for u in $1..n$:

 for v in $1..n$:

$$d[u, v] = \min(d[u, v], d[u, k] + d[k, v])$$

careful w/ Δ !

$\Delta + \Delta$ could overflow?

$\Theta(n^3)$!

- Vertex u is part of a negative cycle iff $d[u, u] < 0$.

So just check $d[u, u]$ for all vertices u . If there are any negative cycles, report error (or can do extra work to figure out correct shortest paths).

To keep track of actual shortest paths, let

$\text{next}[u, v] = \text{next vertex after } u \text{ along a shortest path from } u \rightarrow v$.

• Initialize:

- $\text{next}[u, u] = u$.

- $\text{next}[u, v] = v$ if there is an edge $u \rightarrow v$.

- $\text{next}[u, v] = \text{undefined}$ if no edge $u \rightarrow v$.

• To update during inductive step:

- if $d[u, v, k] = d[u, v, k-1]$, $\text{next}[u, v]$ does not change.

- if $d[u, v, k] = d[u, k, k-1] + d[k, v, k-1]$, then

 Set $\text{next}[u, v] = \text{next}[u, k]$.