

Asymptotic Analysis

$$\underline{3n^2 + 2n - 10} \text{ is } \Theta(n^2).$$

$$\frac{n^3 - 5}{n} = n^2 - \frac{5}{n} \text{ is } \Theta(n^2).$$

$$\frac{n^3 - 5}{\sqrt{n}} = n^{2.5} - \frac{5}{\sqrt{n}} \text{ is } \Omega(n^2). \quad \Omega \quad \Omega$$

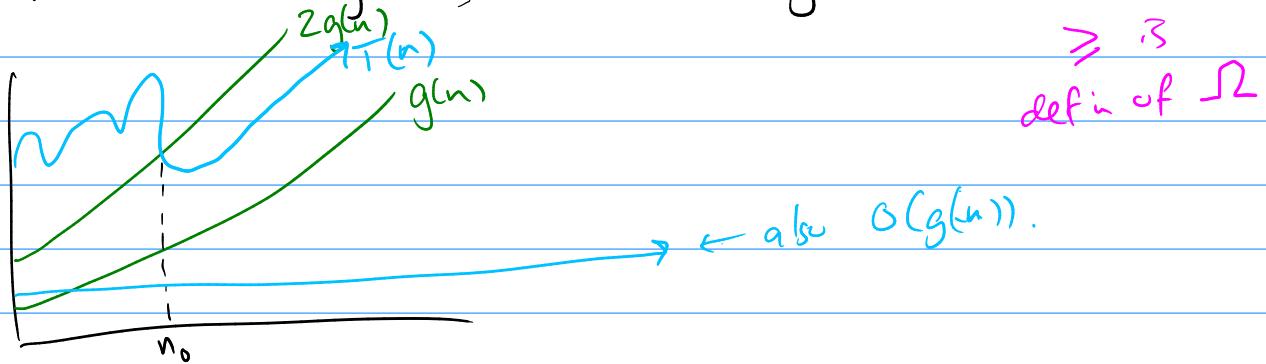
$$n + n\sqrt{n} = n + n^{1.5} \text{ is } O(n^2).$$

$$n^2 \cdot \log_2 n \text{ is } \Omega(n^2).$$

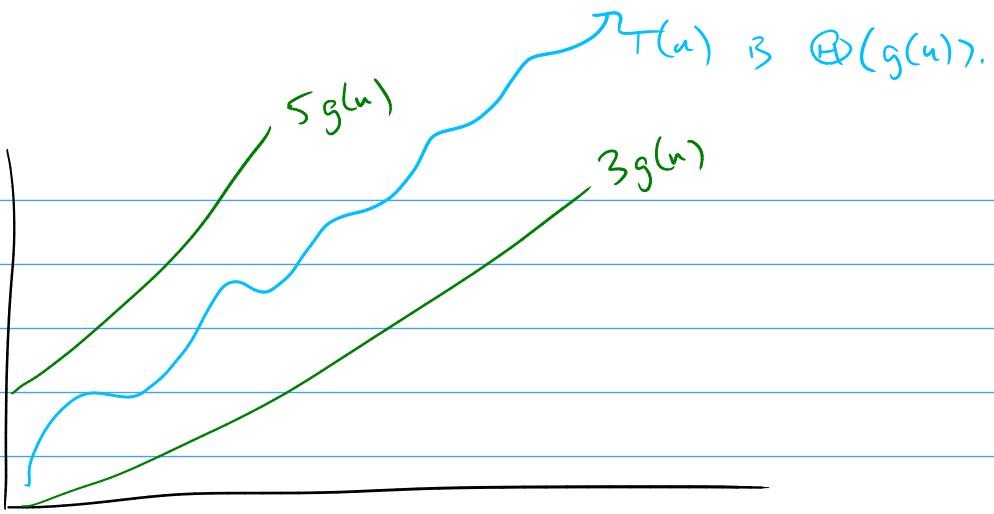
Why do asymptotic analysis?

- Faster than actually running + measuring.
- Insulate ourselves from messy real-world details.
- Works well in practice!

Defin If there exists a real constant $c > 0$ and $n_0 \geq 0$ such that for all $n \geq n_0$, $T(n) \leq c \cdot g(n)$, then we say $T(n)$ is $O(g(n))$.



Defin If $T(n)$ is $O(g(n))$ and $\Omega(g(n))$, then $T(n)$ is $\Theta(g(n))$.



e.g. $\underline{3n^2 + 7n - 5}$

Thm If $0 \leq \lim_{n \rightarrow \infty} \frac{T(n)}{g(n)} < \infty$, then $T(n)$ is $O(g(n))$.

$0 < \dots \leq \infty$, then $T(n)$ is $\Omega(g(n))$

$0 < \dots < \infty$, then $T(n)$ is $\Theta(g(n))$.

Defn If $\lim_{n \rightarrow \infty} \frac{T(n)}{g(n)} = 0$, then we say $T(n)$ is $\circ(g(n))$.
"little-o"

Algorithm with big-O (similar for Ω and Θ)

- $k \cdot O(g(n)) = O(g(n))$ (running something $k \times$)
- $O(f(n)) + O(g(n)) = O(\max(f(n), g(n)))$ (running algorithms in series)
- $O(f(n)) \times O(g(n)) = O(f(n) \times g(n))$ (running nested algorithms)